Color Instabilities in Quark-Gluon Plasma

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30 years

1) St. Mrówczyński,
Stream instabilities of the quark-gluon plasma,
Physica Letters B **214**, 587 (1988), Erratum B **656**, 273 (2007)

2) St. Mrówczyński,
Plasma Instability at the initial stage of ultrarelativistic heavy-ion collisions,
Physics Letters B **314**, 118 (1993)

⋮

5) St. Mrówczyński and M. Thoma,
Hard loop approach to anisotropic systems,
Physical Review D **62**, 036011 (2000)

⋮

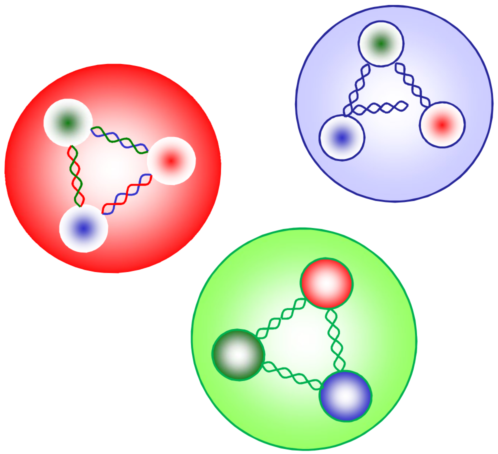
17) St. Mrówczyński, B. Schenke and M. Strickland,
Color instabilities in the quark-gluon plasma,
Physics Reports **682**, 1 (2017)

Elementary Physics Story on Color Instabilities in Quark-Gluon Plasma

Hadrons, Quarks & Gluons

baryons

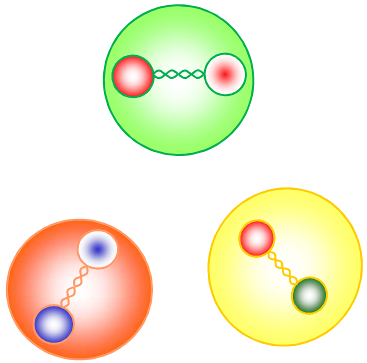
$n, p, \Delta, N^*, \Lambda, \Sigma, \Xi, \Omega, \ldots$



(q, q, q)

mesons

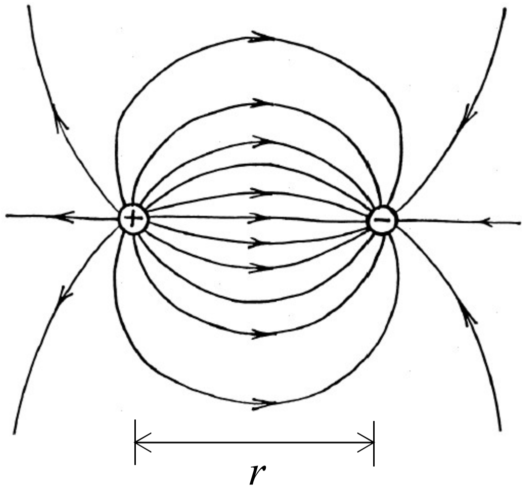
$\pi, K, \rho, \eta, \ldots$



$(q, \overline{q})$

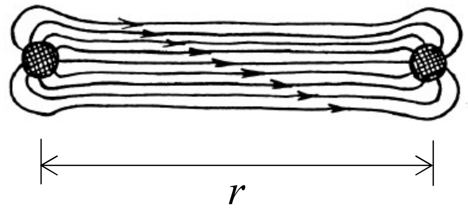
Confinement

Electrodynamics



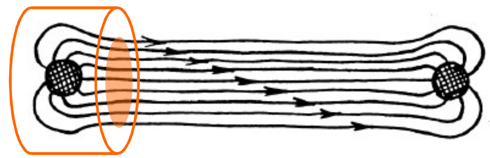
$$E(r) = \frac{e}{r^2} \quad \Rightarrow \quad V(r) = -\frac{e^2}{r}$$

Chromodynamics



$$D = \varepsilon E \quad \Rightarrow \quad \varepsilon = \infty$$

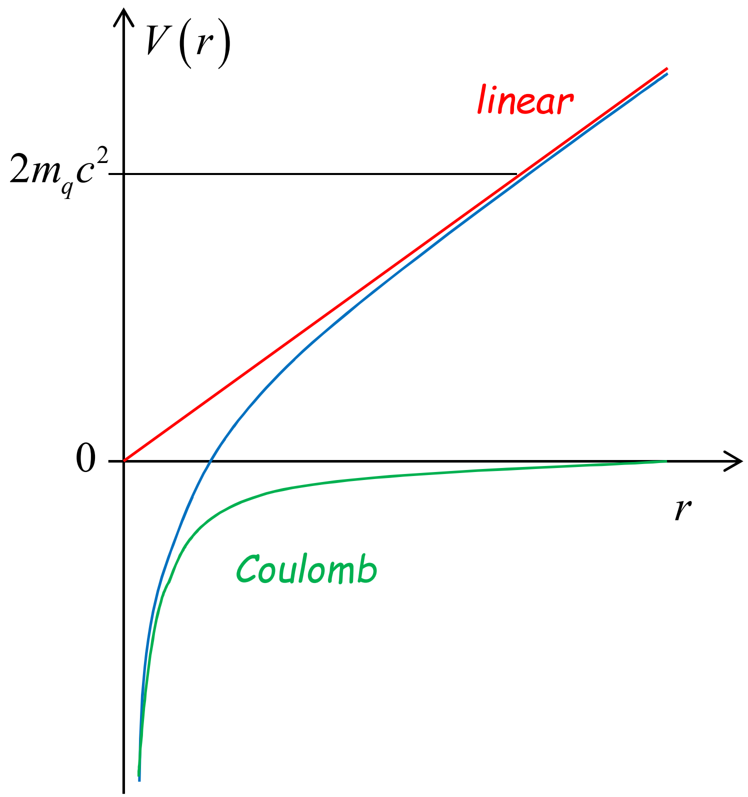
Gauss law



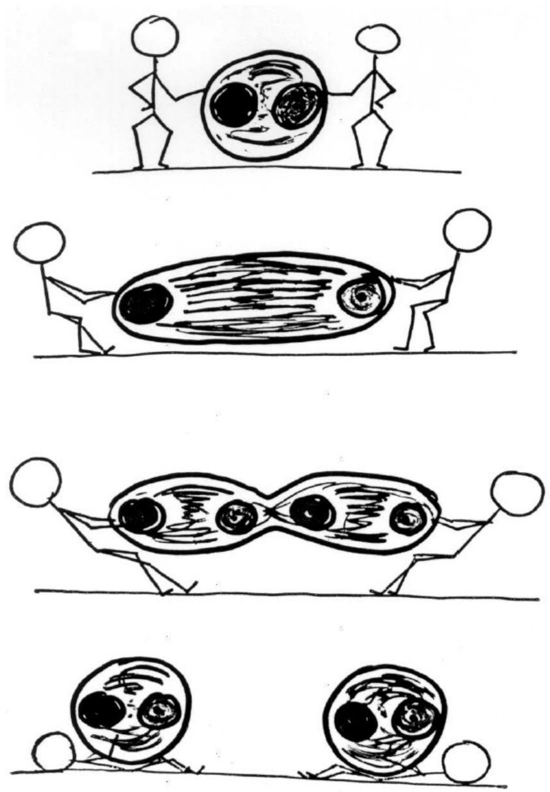
$$\Phi = \sigma E = 4\pi g \qquad \sigma = \text{const}$$

$$E(r) = \frac{4\pi g}{\sigma} \quad \Rightarrow \quad V(r) = \frac{4\pi g}{\sigma} r$$

Confinement cont.



The potential is studied
in spectroscopy of quarkonia.



Asymptotic Freedom

Color charge vanishes at small distances

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)\ln\left(\frac{Q^2}{\Lambda^2_{\text{QCD}}}\right)}$$

Sourceless Maxwell equations in a medium

$$\left\{\begin{array}{l} \nabla\cdot\mathbf{D}=0 \\ \nabla\cdot\mathbf{B}=0 \\ \nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} \\ \nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} \end{array}\right. \quad \Rightarrow \quad \left\{\begin{array}{l} \mathbf{D}=\varepsilon\mathbf{E} \qquad \mathbf{B}=\mu\mathbf{H} \\ \left(\Delta-\frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=0 \\ \left(\Delta-\frac{\varepsilon\mu}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{B}=0 \end{array}\right.$$

$\frac{c}{\sqrt{\varepsilon\mu}}$ phase velocity of EM wave

In vacuum $\varepsilon\mu = 1$

Asymptotic Freedom cont.

$$\varepsilon\mu = 1 \quad \begin{cases} \text{diamagnetic} \quad \mu < 1 \quad \Rightarrow \quad \text{dielectric} \quad \varepsilon > 1 \\ \text{paramagnetic} \quad \mu > 1 \quad \Rightarrow \quad \text{paraelectric} \quad \varepsilon < 1 \end{cases}$$

diamagnetic $\mu < 1$ $\Rightarrow$ dielectric $\varepsilon > 1$

Charges are screened

paramagnetic $\mu > 1$ $\Rightarrow$ paraelectric $\varepsilon < 1$

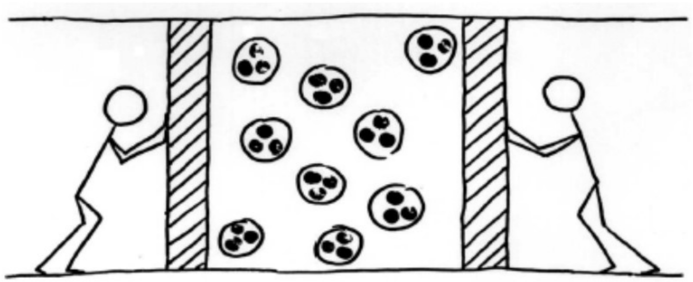
Charges are antiscreened!

Quarks as fermions produce diamagnetic effect

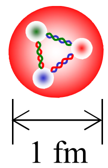
Gluons as bosons produce paramagnetic effect

Gluons win!

Creation of Quark-Gluon Plasma



compression of nuclear matter



$\rho_0 = 0.12\ \text{fm}^{-3}$

normal nuclear density

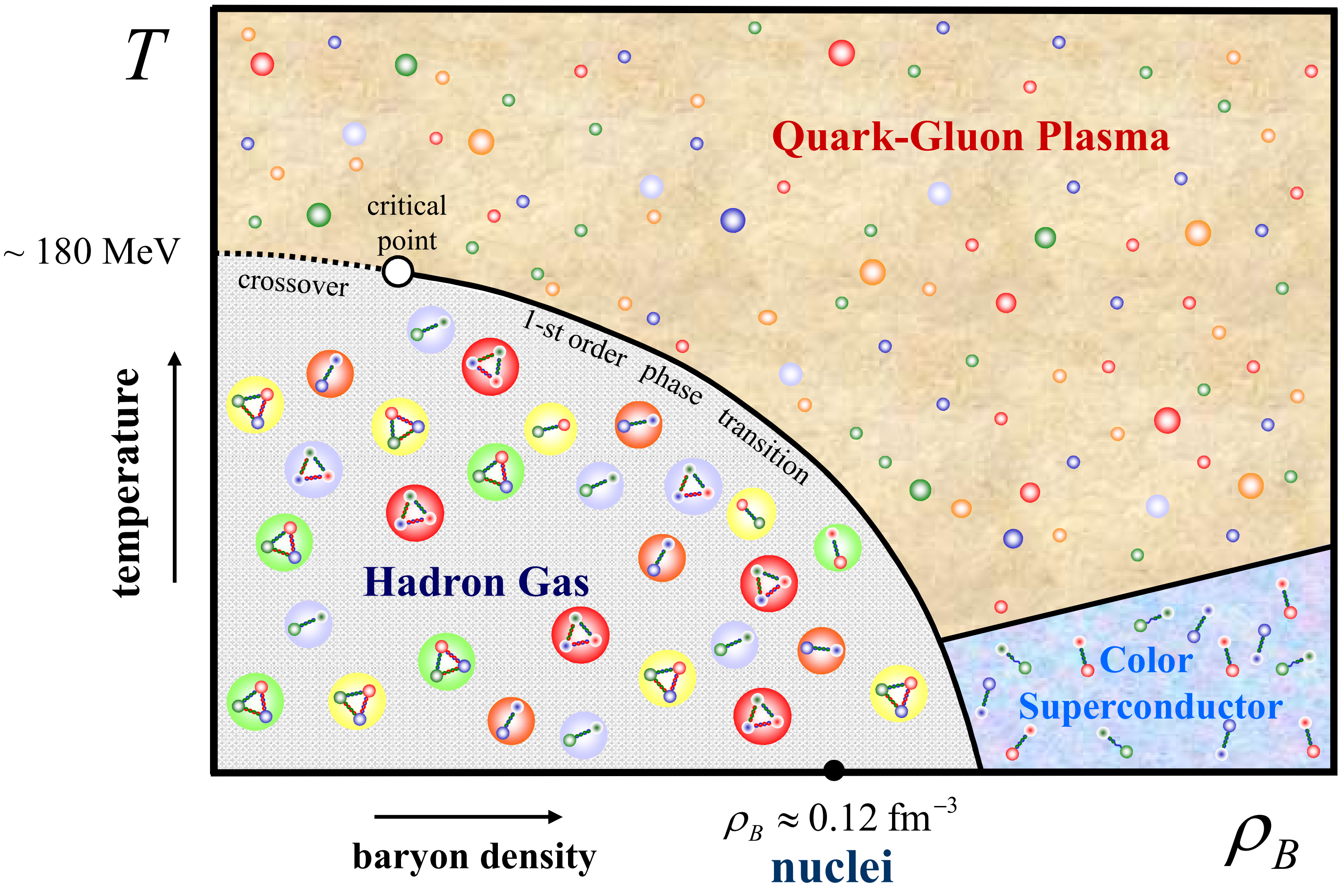
heating up hadron gas

hadron density $\rho \sim T^3$

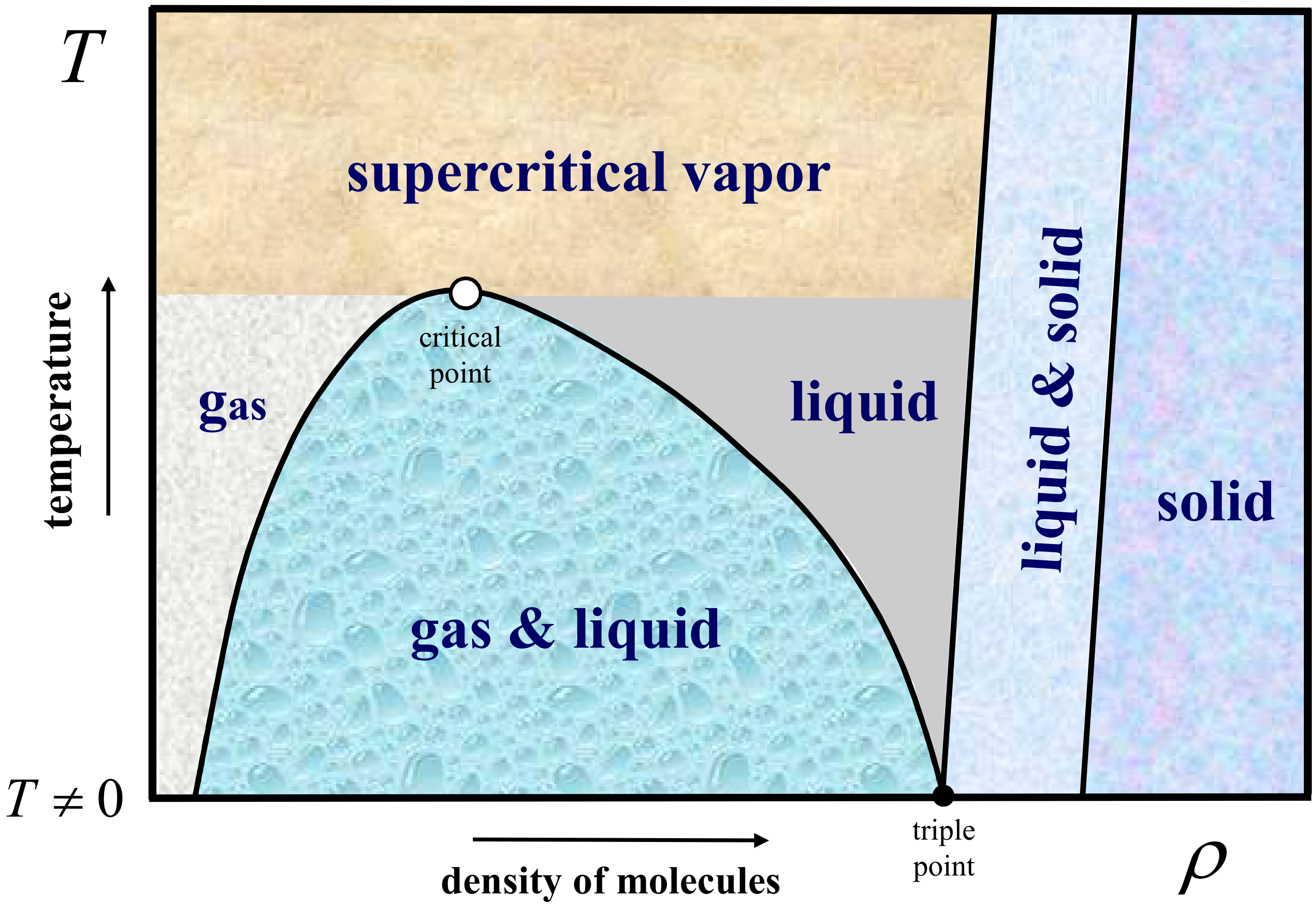
$m_\pi << T$

Natural system of units: $\hbar = c = k_B$

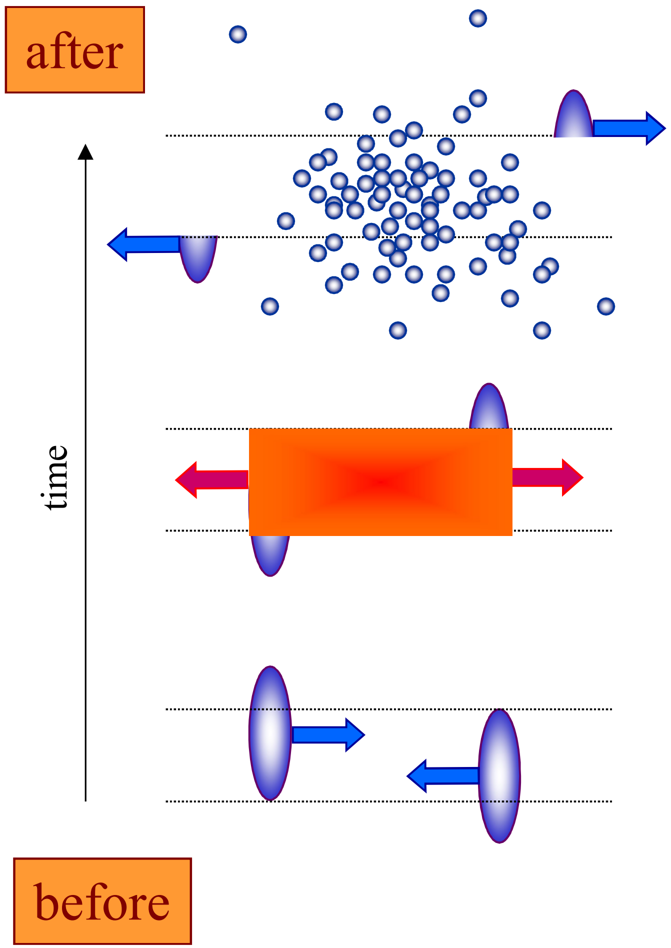
Phase diagram of strongly interacting matter

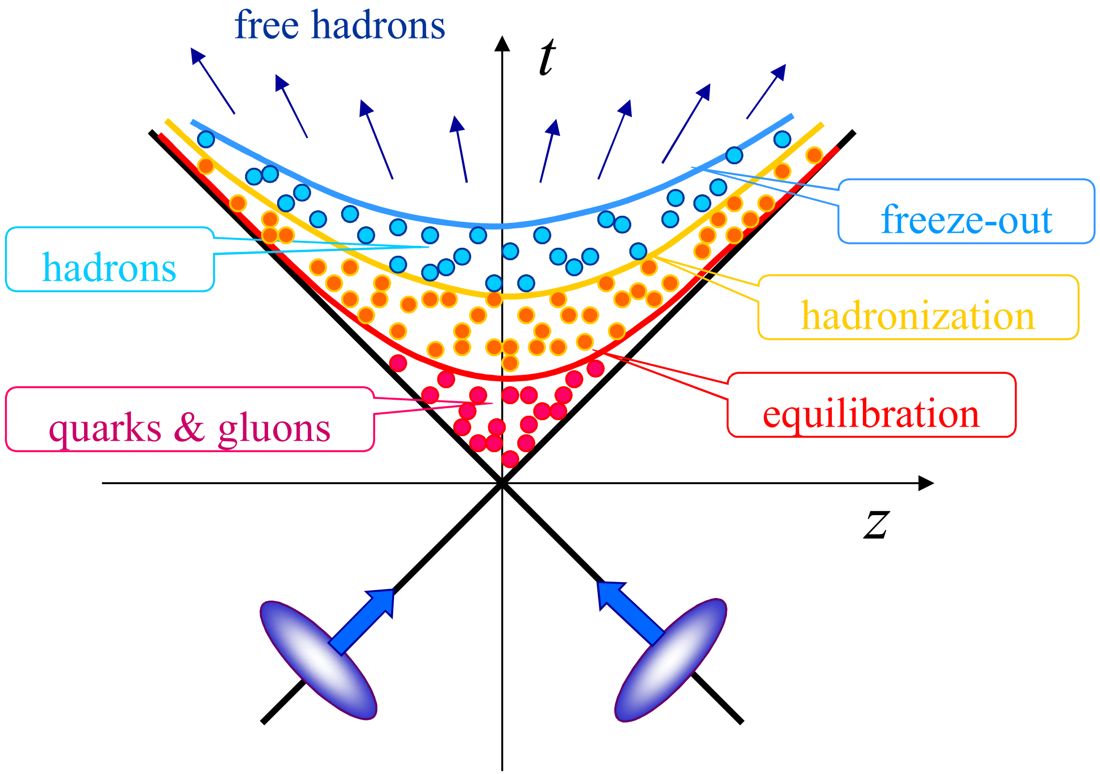


Schematic phase diagram of a simple fluid



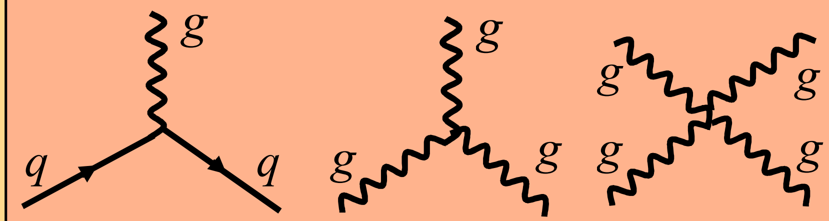
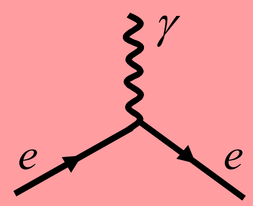
Relativistic heavy-ion collisions





An important role of boost invariance $\tau = \sqrt{t^2 - z^2}$

Quark-Gluon Plasma vs. EM Plasma

		Quark-Gluon Plasma	Electromagnetic Plasma
Underlying Microscopic Theory		QCD	QED
Elementarny Interactions			
Constituents	Fermions	quarks, antiquarks	electrons, positrons
	Massless Gauge Bosons	gluons	photons
		-	massive ions
Coupling		$\alpha(Q^2) = \frac{g^2}{4\pi} \approx 0.1 - 1$	$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$

Ultrarelativistic Quark-Gluon Plasma

Plasma constituents – quarks & gluons – are massless!

$$m_q << T$$

Temperature T is often the only dimensional parameter.

density: $\rho \sim T^3$

inter-particle spacing: $l \sim T^{-1}$

energy density: $\varepsilon \sim T^4$

pressure: $p \sim T^4$

Weakly Coupled Quark-Gluon Plasma

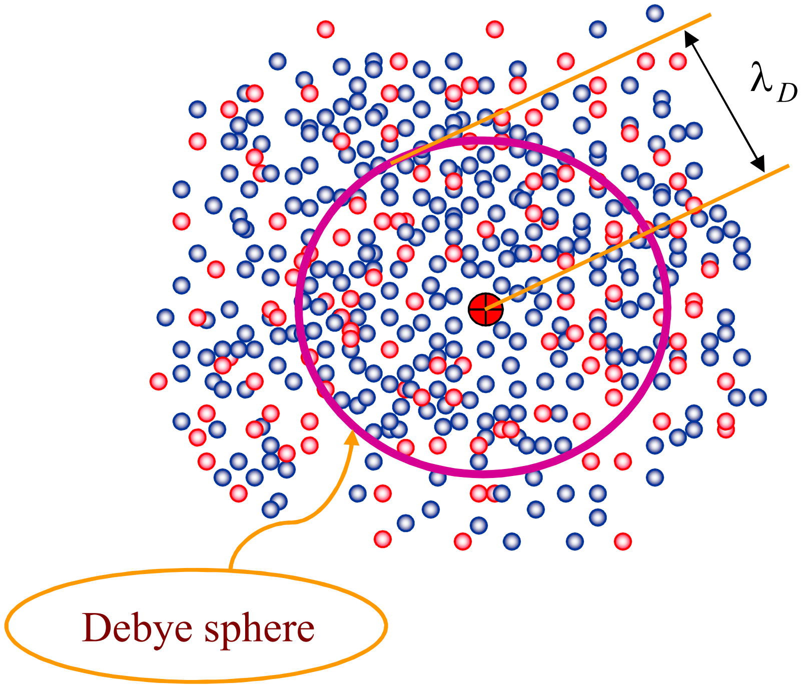
Plasma from the earliest stage of relativistic heavy-ion collisions
is assumed to be weakly coupled.

Asymptotic freedom formula: $$\alpha_S(Q^2) = \frac{12\pi}{(33-2N_f)\ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)}$$

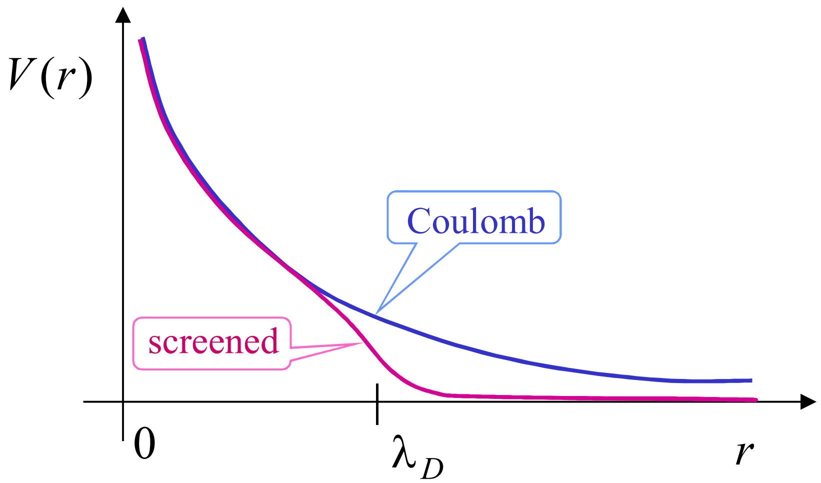
Dimensional argument: $$Q \rightarrow \varepsilon^{1/4}$$

ε - energy density

Plasma manifests collective behavior



$\lambda_D = \frac{1}{m_D} \sim \frac{1}{gT}$ screening length $V(r) \sim \frac{e^{-\frac{r}{\lambda_D}}}{r}$



$$V_D = \frac{4}{3}\pi\lambda_D^3 \sim \frac{1}{g^3T^3}, \qquad n \sim T^3, \qquad n\,V_D \sim \frac{1}{g^3} >> 1 \quad \text{if} \quad g << 1$$

In a weakly coupled plasma, there are many particles in a Debye sphere!

Screening length

Poisson equation

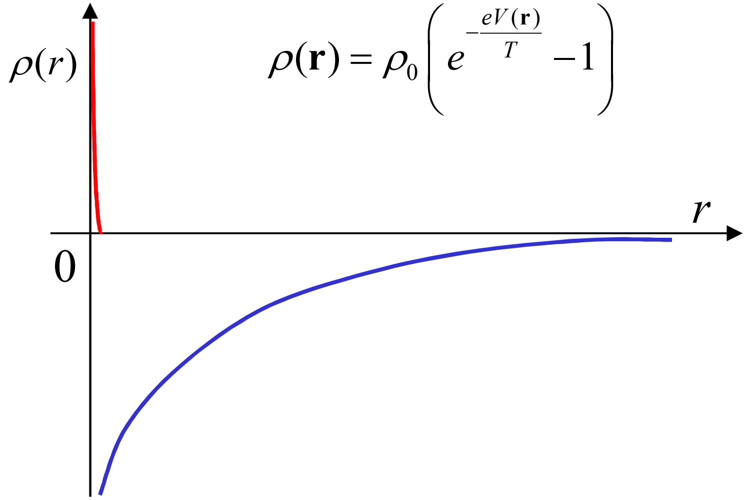
$$\Delta V(\mathbf{r}) = -e\rho(\mathbf{r})$$

$$e|V(\mathbf{r})| << T$$

$$e^{-\frac{eV(\mathbf{r})}{T}} - 1 \approx 1 - \frac{eV(\mathbf{r})}{T} \ldots - 1 = -\frac{eV(\mathbf{r})}{T}$$

$$\Delta V(\mathbf{r}) = -e\rho(\mathbf{r}) \approx \frac{e^2\rho_0}{T} V(\mathbf{r})$$

charge density



$$\rho(\mathbf{r}) = \rho_0\left(e^{-\frac{eV(\mathbf{r})}{T}} - 1\right)$$

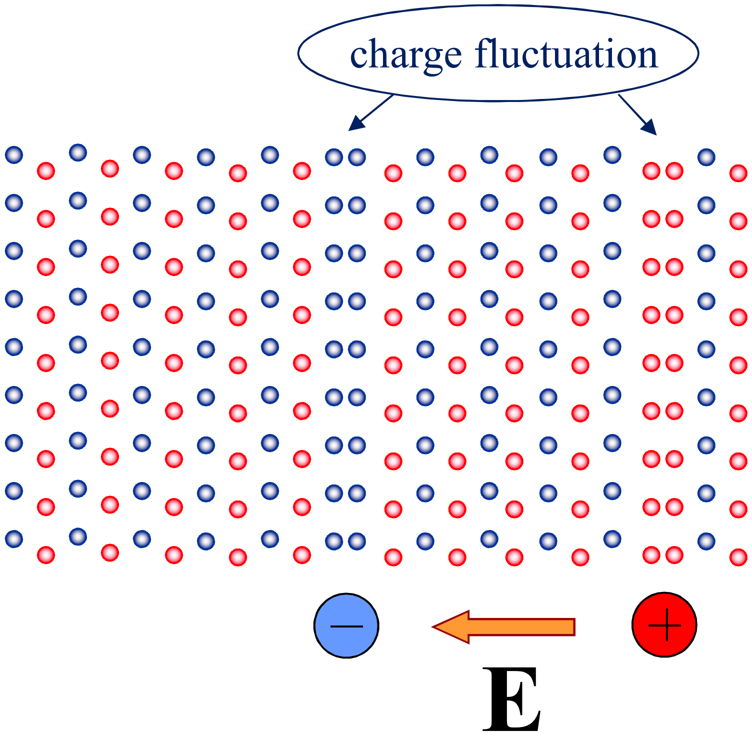
$$\frac{d^2}{dx^2}V(x) = m_D^2 V(x) \quad \Rightarrow \quad V(x) \sim e^{-m_D|x|}$$

Debye mass

$$m_D \equiv e\sqrt{\frac{\rho_0}{T}} = \frac{1}{\lambda} \sim eT$$

$$\rho_0 \sim T^3$$

Plasma oscillations

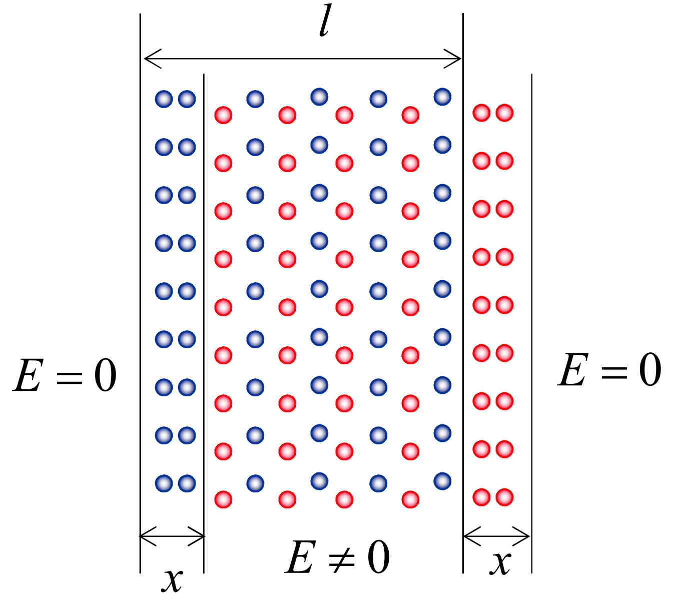


$$\mathbf{E}(t,\mathbf{r}) = \mathbf{E_0}\cos(\omega(\mathbf{k})t - \mathbf{k}\cdot\mathbf{r} + \varphi)$$

$$\omega(\mathbf{k}) \underset{\mathbf{k}\to 0}{\approx} \omega_p \sim gT$$

plasma or Langmuir frequency

Plasma frequency



Gauss theorem

$$\Phi = Q_s$$

Flux $\Phi = ES$

Charge $Q_s = e\rho Sx$

Electric field $E = e\rho x$

Equation of motion

$$M\ddot{x} = F$$

Mass $M = \rho mSl$

Force $F = QE$

Charge $Q = e\rho Sl$

Harmonic oscillator

$$\ddot{x} = -\omega_p^2 x$$

plasma frequency

$$\omega_p \equiv e\sqrt{\frac{\rho}{m}}$$

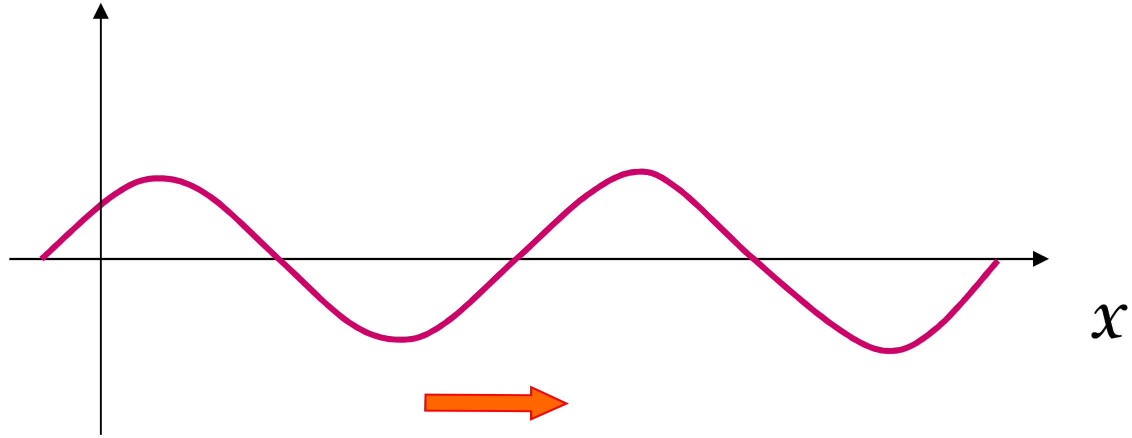
$$\lambda \to \infty$$

Qurak-gluon plasma

$$e \to g$$
$$\rho \sim T^3$$
$$m \sim T$$
$$\omega_p \sim gT$$

Landau damping

$$E^x(t,x) = E_0 \cos(\omega_0 t - kx)$$



$$v_\varphi = \frac{\omega_0}{k}$$

Resonance energy transfer from electric field to particles with $v = v_\varphi$

Instabilities

stationary state

$$A(t) = A_0 + \delta A(t)$$

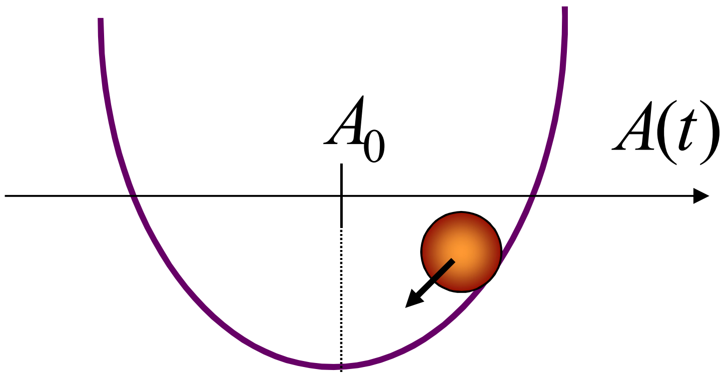
fluctuation

Instability

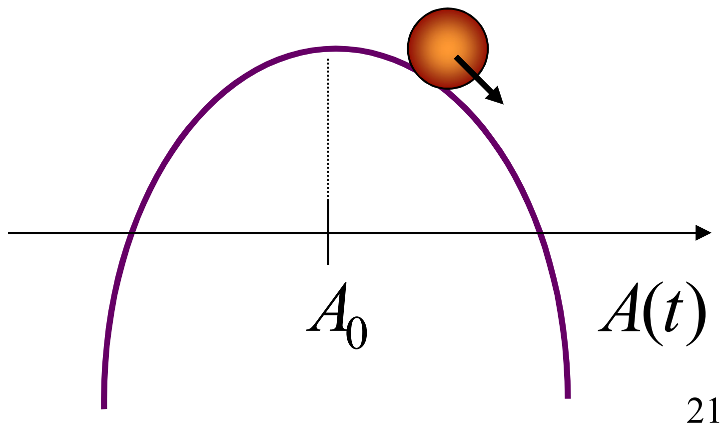
$$\delta A(t) \propto e^{\gamma t}$$

$$\gamma > 0$$

stable configuration



unstable configuration



Plasma instabilities

- **instabilities in configuration space – hydrodynamic instabilities**

- **instabilities in momentum space – kinetic instabilities**

instabilities due to non-equilibrium momentum distribution

$$f(\mathbf{p}) \text{ is not } \sim \exp\left(-\frac{E}{T}\right)$$

Kinetic instabilities

- longitudinal modes – $\mathbf{k} \parallel \mathbf{E}, \;\; \delta\rho \sim e^{-i(\omega t - \mathbf{kr})}$

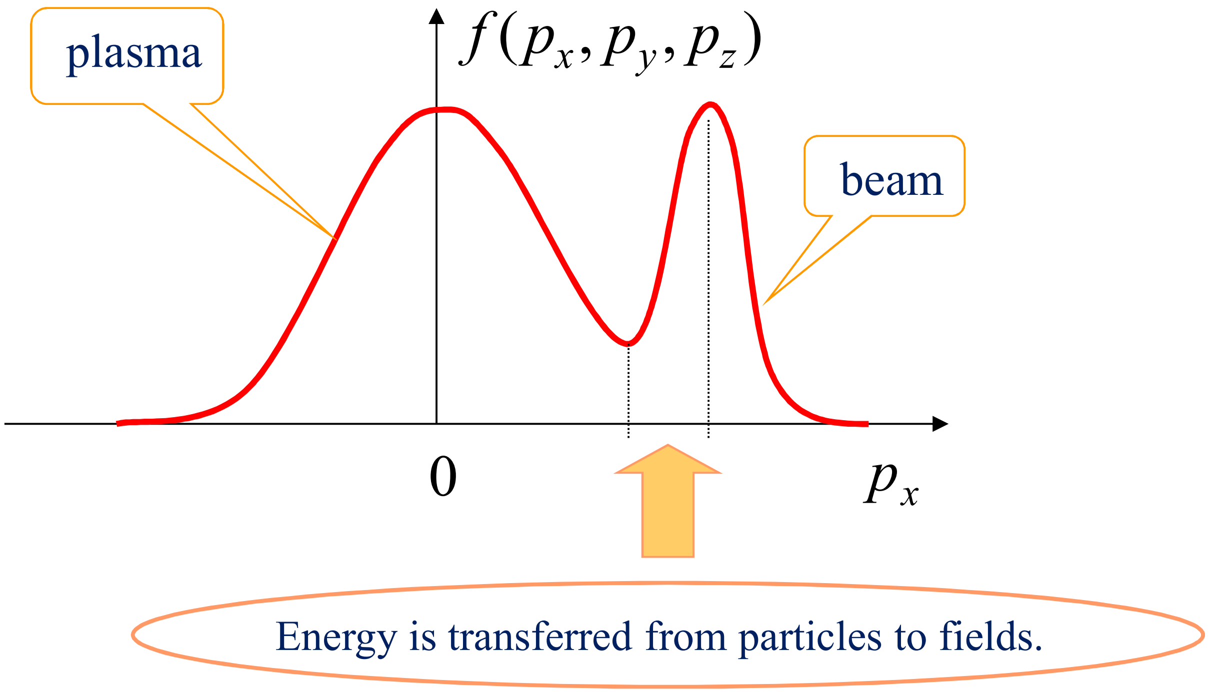
- transverse modes – $\mathbf{k} \perp \mathbf{E}, \;\; \delta\mathbf{j} \sim e^{-i(\omega t - \mathbf{kr})}$

E – electric field, **k** – wave vector, ρ – charge density, **j** - current

Which modes are relevant for QGP
from relativistic heavy-ion collisions?

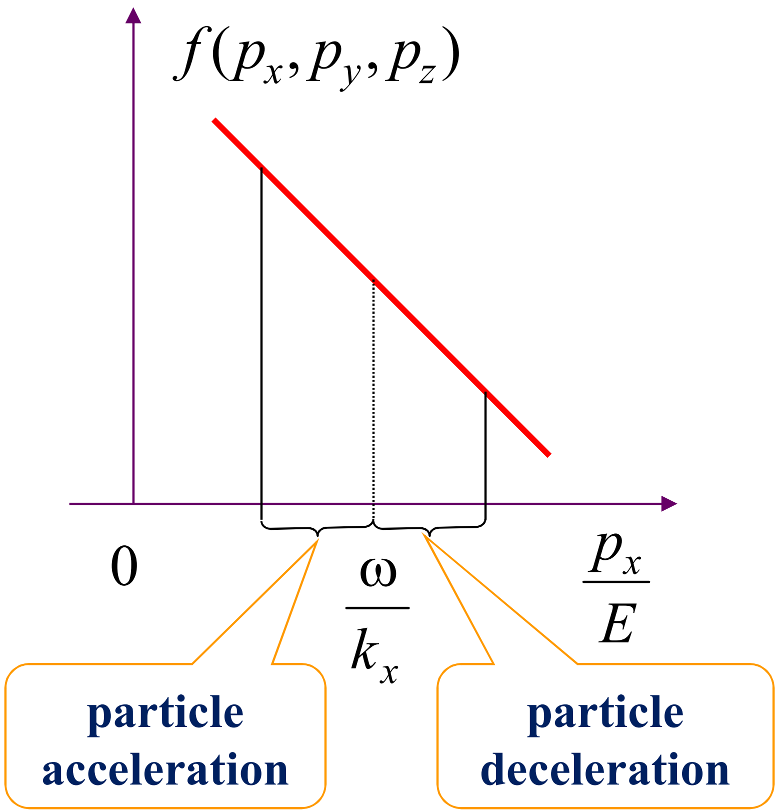
Logitudinal modes

unstable configuration

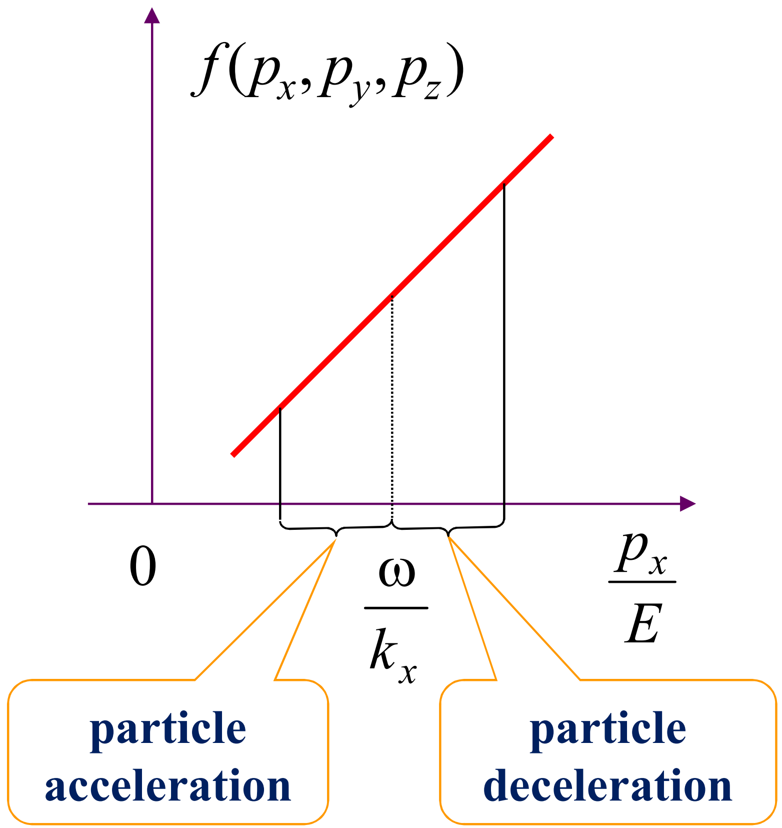


Logitudinal modes

Electric field decays - damping

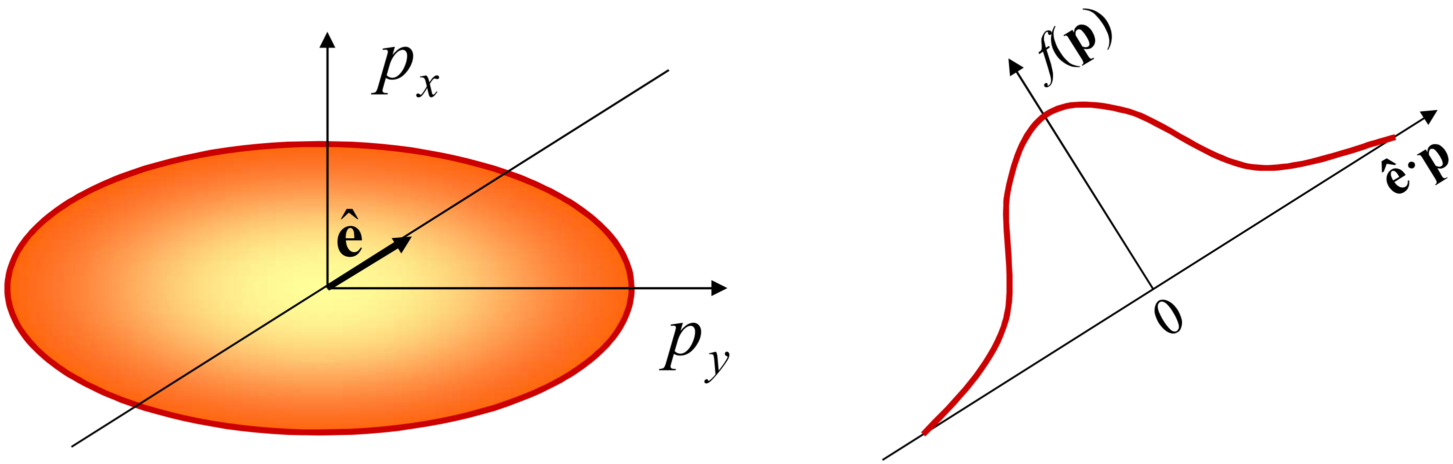


Electric field grows - instability



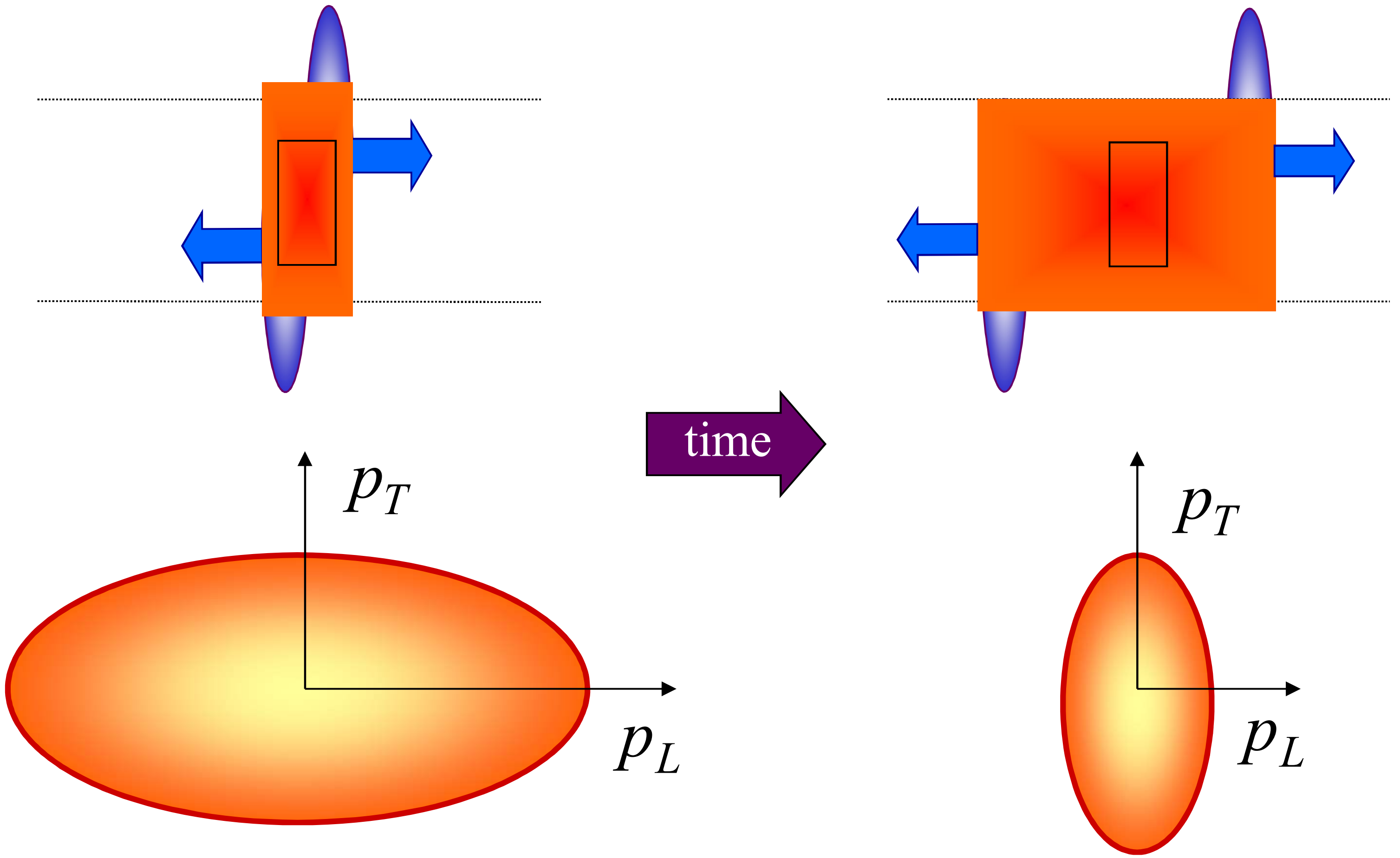
$\frac{\omega}{k_x}$ - phase velocity of the electric field wave, $\frac{p_x}{E}$ - particle's velocity

Parton momentum distribution in AA collisions



- Momentum distribution has a single maximum and monotonously decreases in every direction.
- Longitudinal unstable modes are irrelevant for relativistic heavy-ion collisions.
- There are unstable transverse modes.

Evolution of Parton Momentum Distribution



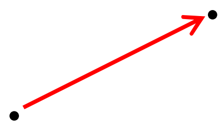
prolate

oblate

Seeds of instability

$\langle j_a^{\mu}(x) \rangle = 0$ but current fluctuations are finite

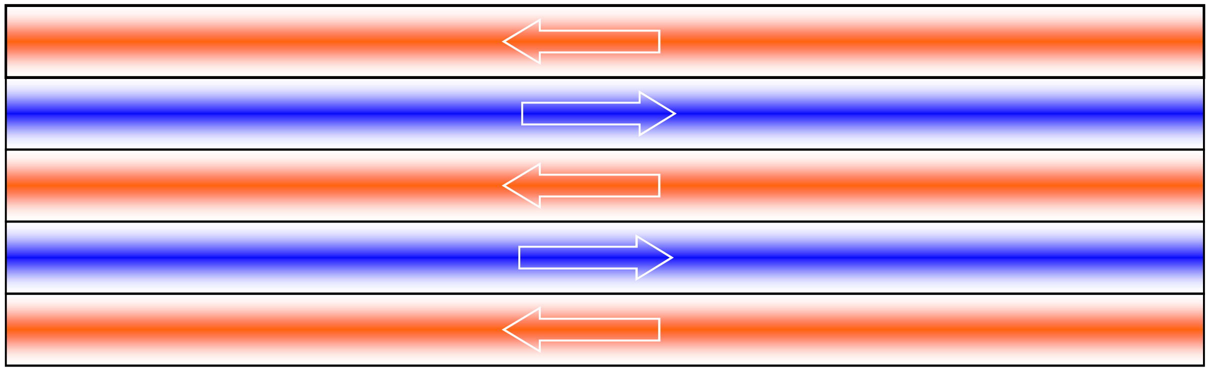
$$\left\langle j_a^{\mu}(x_1)\, j_b^{\nu}(x_2) \right\rangle = \frac{1}{2}\delta^{ab} \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu} p^{\nu}}{E_p^2} f(\mathbf{p}) \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \neq 0$$



$x_2 = (t_2, \mathbf{x}_2)$

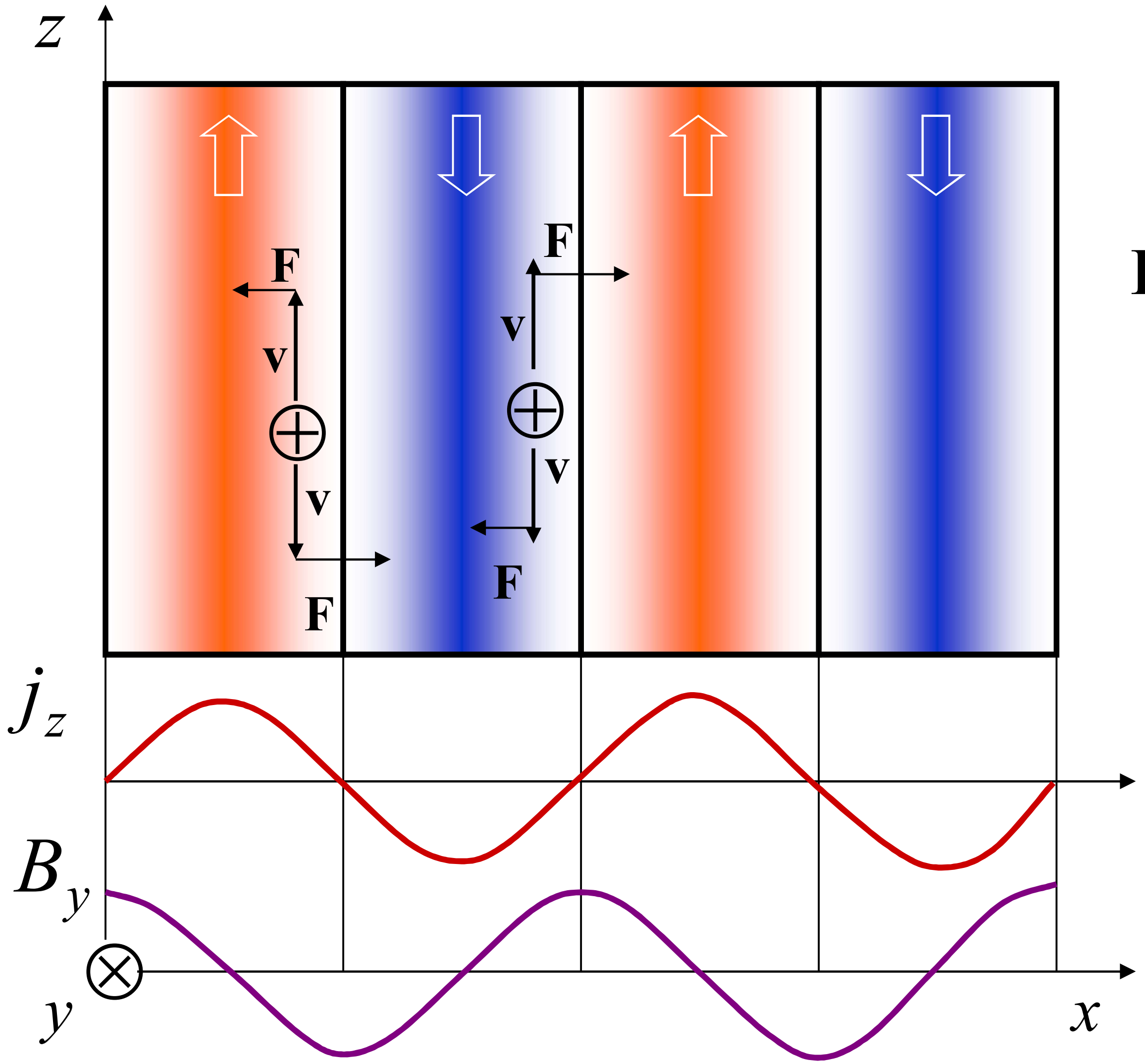
$x_1 = (t_1, \mathbf{x}_1)$

$x \equiv (t_1 - t_2, \mathbf{x}_1 - \mathbf{x}_2)$



Direction of the momentum surplus

Mechanism of filamentation



Lorentz force

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Ampere's law

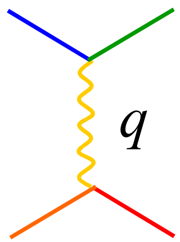
$$\nabla \times \mathbf{B} = \mathbf{j}$$

Time scale & collisional damping

Time scale of collective phenomena

$$t_{\text{collec}} \sim \frac{1}{g\,T} \;\Rightarrow\; \nu_{\text{collec}} \sim \frac{1}{t_{\text{collec}}} \sim gT$$

Parton-parton scattering



hard scattering: $q \sim T$

soft scattering: $q \sim gT$

Frequency of collisions

$$\nu_{\text{hard}} \sim g^4 \ln(1/g)\,T$$

$$\nu_{\text{soft}} \sim g^2 \ln(1/g)\,T$$

$$g^2 << 1 \;\Rightarrow\; \nu_{\text{hard}} << \nu_{\text{soft}} << \nu_{\text{collec}}$$

The instabilities are fast!

Growth of instabilities – 1+1 numerical simulations

SU(2) Hard Loop Dynamics

1+1 dimensions

$$A_a^\mu = A_a^\mu(t,z)$$

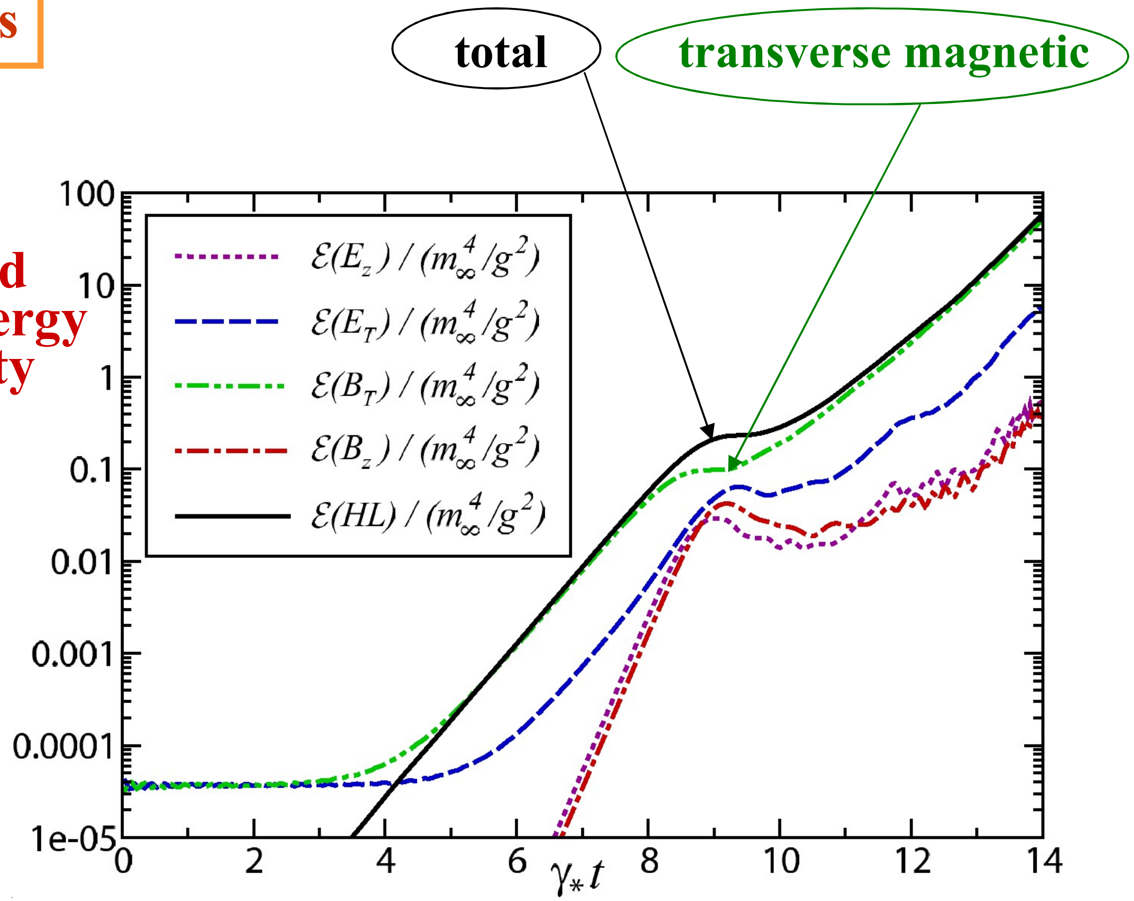
Anisotropic particle's momentum distribution

$$f(\mathbf{p}) = f_{\text{iso}}(|\mathbf{p}| + \zeta p_z)$$

$$m_D^2 = -\frac{\alpha_s}{\pi}\int_0^\infty dp\, p^2 \frac{df_{\text{iso}}(p)}{dp}$$

$$(m_D, \zeta)$$

Scaled field energy density



Strong anisotropy $\zeta = 10$ γ_* - maximal growth rate

A. Rebhan, P. Romatschke & M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005)

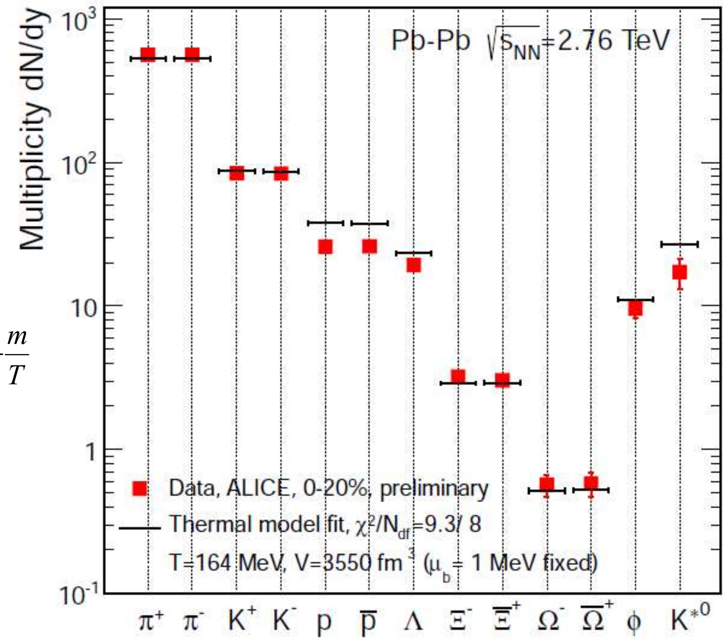
What is the role of instabilities in nuclear collisions?

Instabilities speed up equilibration of quark-gluon plasma

Thermodynamic equilibrium in nuclear collisions

Chemical equilibrium

$$\sim e^{-\frac{m}{T}}$$

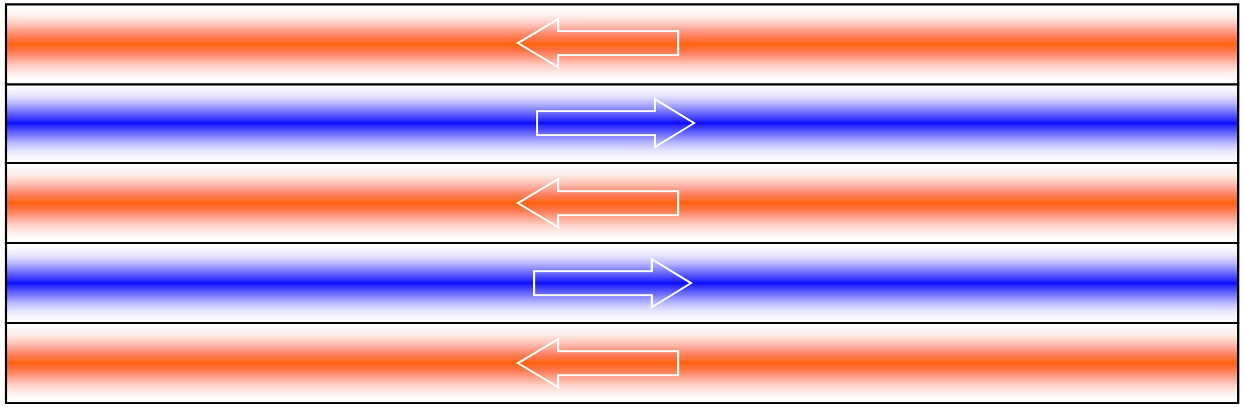


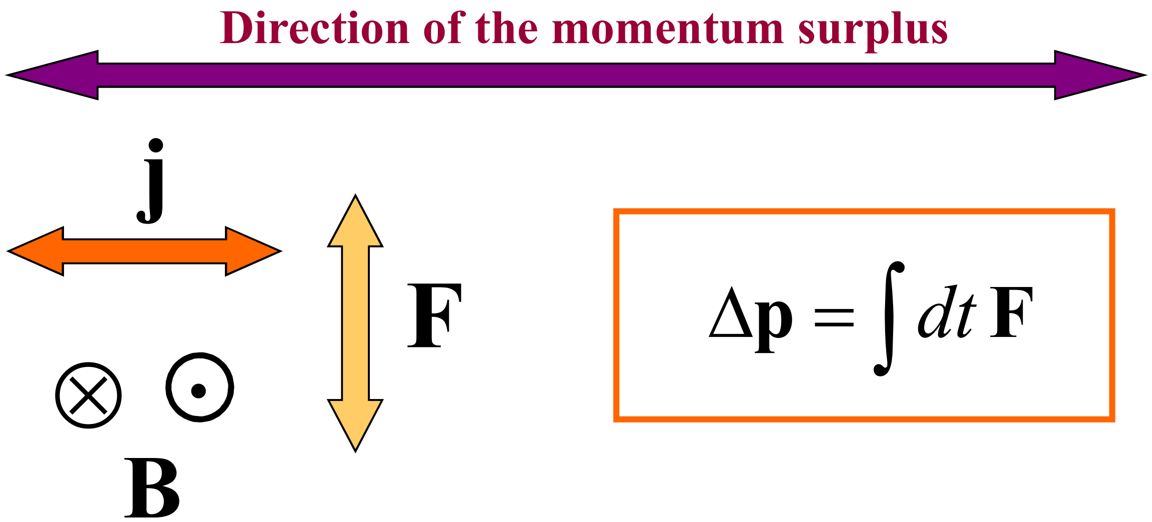
A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nuclear Physics A **904-905**, 535c (2013)

Kinetic equilibrium

- particle spectra
- collective flow

Isotropization - particles





$$\Delta \mathbf{p} = \int dt\, \mathbf{F}$$

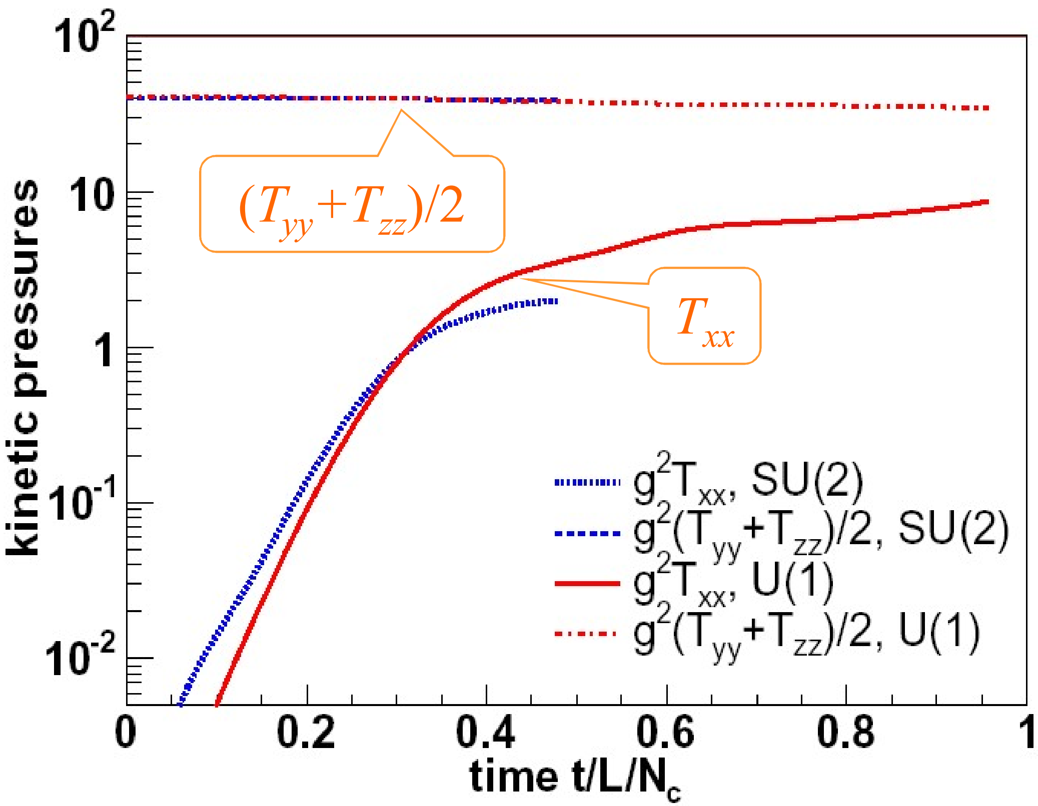
Isotropization – numerical simulation

Classical system of colored particles & fields

$$T_{ij} = \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{E} f(\mathbf{p})$$

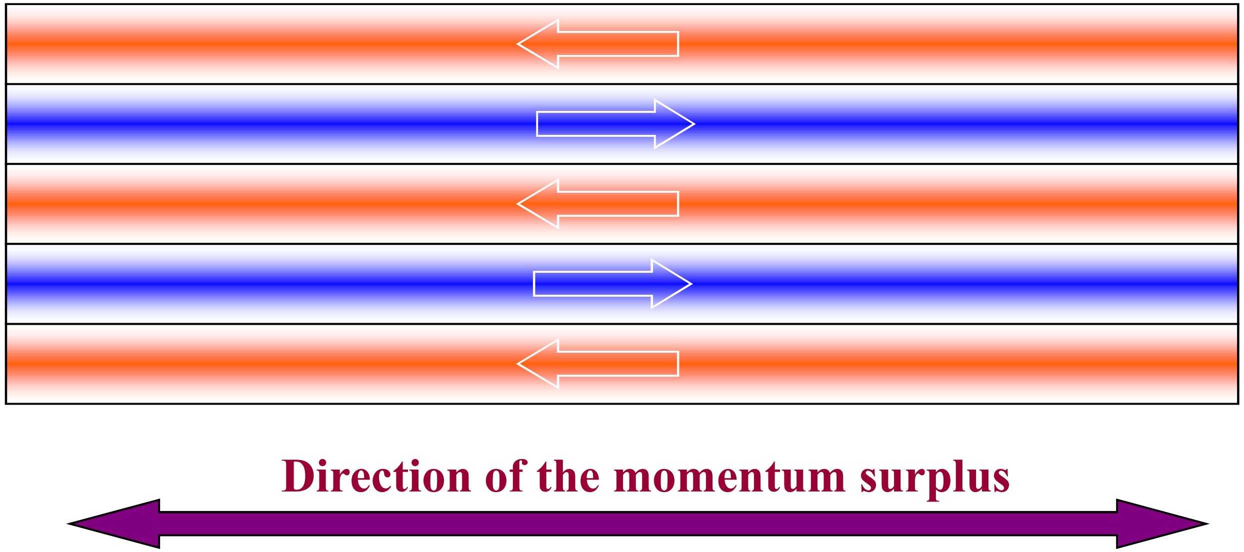
Isotropy:

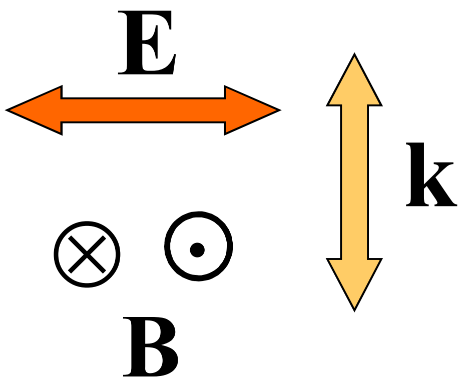
$$T_{xx} = (T_{yy} + T_{zz}) / 2$$



A. Dumitru & Y. Nara, Phys. Lett. **B621**, 89 (2005).

Isotropization - fields





$$\mathbf{P}_{\text{fields}} \sim \mathbf{B}^a \times \mathbf{E}^a \sim \mathbf{k}$$

Conclusions

- Non-equilibrium QGP can be unstable
- Unstable transverse modes are relevant for AA collisions
- Instabilities drive equilibration