Hadron structure functions and the pion distribution amplitude from Lattice QCD

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and the RQCD collaboration

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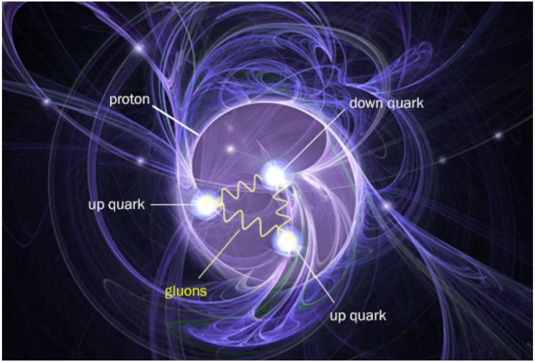
Hadrons' internal structure

Standard Model of elementary particles: electrons, muons, quarks, gluons, photons, $W^{\pm}$, Z, Higgs, ...



Credit: *Brookhaven National Lab website*

Experiment: HERA, LHC, Electron-Ion Collider study hadron structure functions and try to discover the origin of mass,
Theory: Quantum Chromodynamics (QCD) is the theory describing the interations of quarks and gluons.

Quantum Chromodynamics

Lattice Quantum Chromodynamics

- space-time is discretized ⇒ finite dimensional problem fits into a computer,
- equations of QCD are solved numerically,
- the only available *ab initio* approach.

Monte Carlo simulations of Lattice QCD

- *physical observable* = very high dimensional integral,
- Monte Carlo integration with Boltzmann probability distribution,
- Markov chains to generate samples = configurations,
- many different observables can be estimated using one ensemble of configurations,
- Hybrid Monte Carlo algorithm allows global updates.

Outline

- Qualitative estimation of moments of pion distribution amplitude (RQCD collaboration)
- Quantitative estimation of full x-dependence of nucleon PDFs (ETMC collaboration)
- Quantitative estimation of full x-dependence of pion distribution amplitude (RQCD collaboration)

Definition

Pion DA is the quantum amplitude that the pion moving with momentum P is built of a pair of quark and antiquark moving with momentum xP and $(1-x)P$ respectively.

Relevance

Pion photoproduction: two off-shell photons provide the hard scale necessary for the factorization into the perturbative and non-perturbative parts. Transition form factor measured most recently experimetaly by BaBar '09 and Belle '12.

Implementation

2nd moment of the pion DA, $\langle \xi^2 \rangle$, can be obtained numerically from two-point correlation functions.

Pion distribution amplitude

Definition, Braun *et al.*, '15

$$\langle 0|\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n)|\pi(p)\rangle =$$
$$= if_\pi(p\cdot n)\int_0^1 dx e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_\pi(x,\mu^2)$$

Neglecting isospin breaking effects $\phi_\pi(x)$ is symmetric under the interchange of momentum fraction $x \to (1-x)$

$$\phi_\pi(x,\mu^2) = \phi_\pi(1-x,\mu^2)$$

Moments of the momentum fraction difference

$$\xi = x - (1-x)$$

are interesting

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x,\mu^2)$$

$$\phi_\pi(x,\mu^2) = 6u(1-u)\left[1 + \sum_n a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1)\right]$$

Pion distribution amplitude

Local operators, Braun *et al.*, '15

The nonlocal operator can be Taylor expanded and expressed in terms of local operators with derivatives

$$\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \dots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho,\mu_1,\dots,\mu_{i+1}}$$

where

$$\mathcal{M}^{(k,l)}_{\rho,\mu_1,\dots,\mu_{k+l}} = \bar{d}(0)\overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \dots \overrightarrow{D}_{\mu_{k+l}} \gamma_{\rho)}\gamma_5 u(0)$$

Consequently,

$$i^{k+l}\langle 0|\mathcal{M}^{(k,l)}_{\rho,\mu_1,\dots,\mu_{k+l}}|\pi(p)\rangle = i f_\pi p_{(\rho} p_{\mu_1} \dots p_{\mu_{k+l})}\langle x^l (1-x)^k \rangle$$

Lattice operators for the 2nd moment, Braun *et al.*, '15

Two operators local are relevant

$$\mathcal{O}^{-}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

and

$$\mathcal{O}^{+}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

We estimate the following correlation functions

$$C_\rho(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}_\rho(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

$$C^{\pm}_{\rho\mu\nu}(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}^{\pm}_{\rho\mu\nu}(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

Lattice operators for the 2nd moment, Braun *et al.*, '15

From the correlation functions we construct ratios

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$

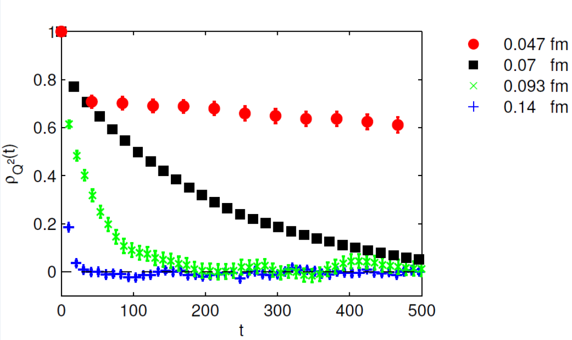
which exhibit plateaux and which we fit to extract the value $R^{\pm}_{\rho\mu\nu,\sigma}$. Finally,

$$\langle\xi^2\rangle^{\overline{\mathrm{MS}}} = \zeta_{11}R^{-} + \zeta_{12}R^{+},$$

$$a_2^{\overline{\mathrm{MS}}} = \frac{7}{12}\left[5\zeta_{11}R^{-} + (5\zeta_{12} - \zeta_{22})R^{+}\right]$$

where ζ_{ij} are renormalization constants estimated nonperturbatively.

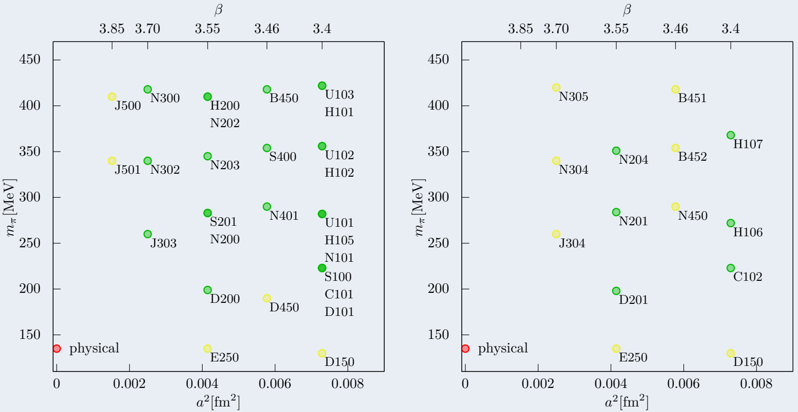
Periodic boundary conditions



Credit: *ALPHA Collaboration, Nucl.Phys. B845 (2011) 93-119*

A very severe critical slowing down of the topological charge in pure Yang-Mills theory has been observed when using the HMC algorithm, implying that the simulations scale as a^{-10}.

Coordinated Lattice Simulations collaboration

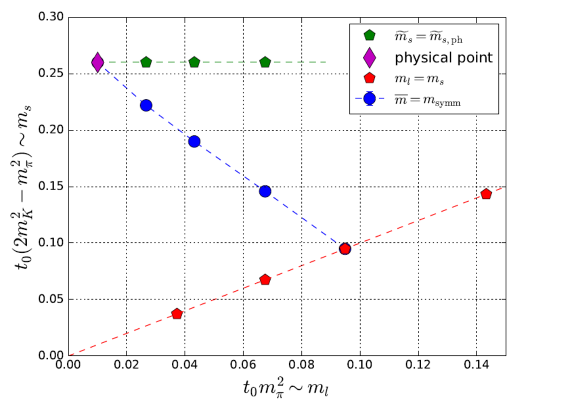


Credit: *J. Simeth, Univ. Regensburg*

CLS

CLS: CERN, DESY, Univ. Regensburg, Univ. Mainz, Univ. Madrid, Univ. Munster, Univ. Odensee, Jagiellonian Univ., Univ. Milano, Univ. Dublin

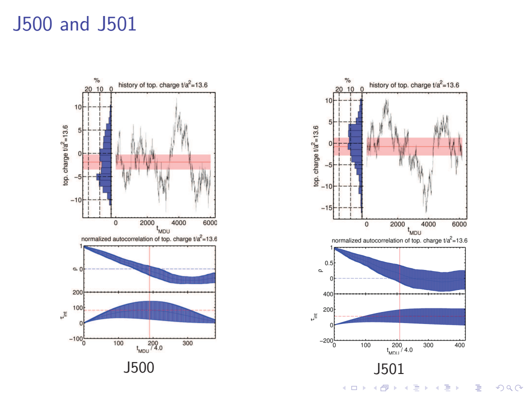
Coordinated Lattice Simulations collaboration



Credit: *W. Söldner, Univ. Regensburg*

Two trajectories lead to the physical point, a third trajectory as generated for non-perturbative renormalization.

Autocorrelation of the topological charge



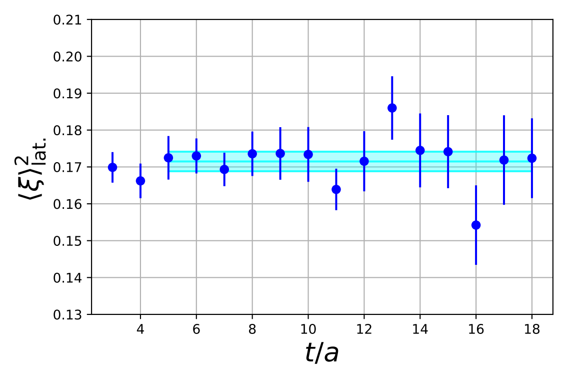
Credit: *J. Simeth, Univ. Regensburg*

In spite of using open boundary conditions we still experience growing autocorrelations times: ~ 200 configurations at $a = 0.039$ fm.

2^{nd} moment of the pion distribution amplitude

Plateau fit example

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$



We use momentum smearing (Bali *et al.* '16) to reduce signal-to-noise problem (Braun *et al.* '17).

2^{nd} moment of the pion distribution amplitude

Combined fit

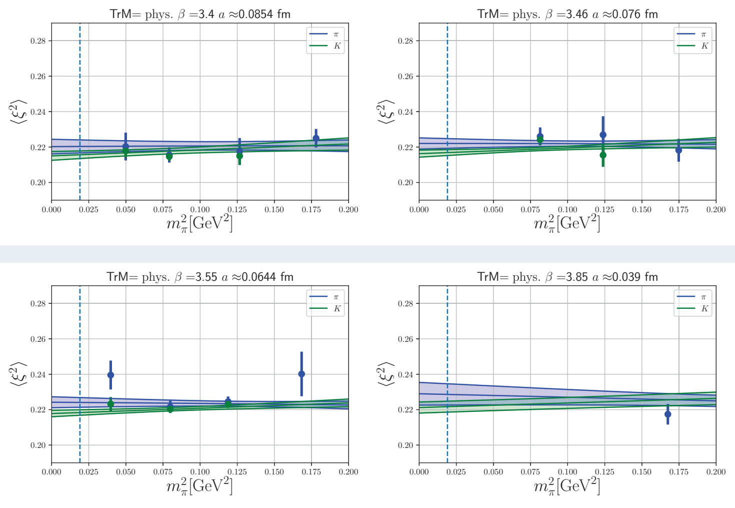
We perform a combined fit to all data points: all lattice spacings and all pion/kaon masses along the three trajectories with the ChPT inspired fit ansatz

$$\overline{M}^2 = \frac{2m_K^2 + m_\pi^2}{3}, \qquad \delta M^2 = m_K^2 - m_\pi^2$$

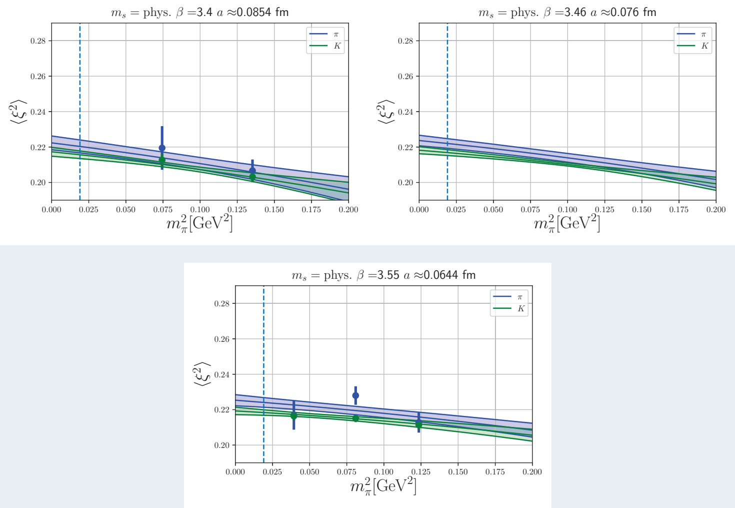
$$\langle \xi^2 \rangle_\alpha = \left(1 + c_0 a + c_1 a \overline{M}^2 + c_2^\alpha a \delta M^2\right) \begin{cases} \langle \xi^2 \rangle_0 + \overline{A}\overline{M}^2 - 2\delta A \delta M^2, & \alpha = \pi, \\ \langle \xi^2 \rangle_0 + \overline{A}\overline{M}^2 + \delta A \delta M^2, & \alpha = K \end{cases}$$

and $\overline{A}$ and δA are combinations of low energy constants.
$\Rightarrow$ 7 fit parameters

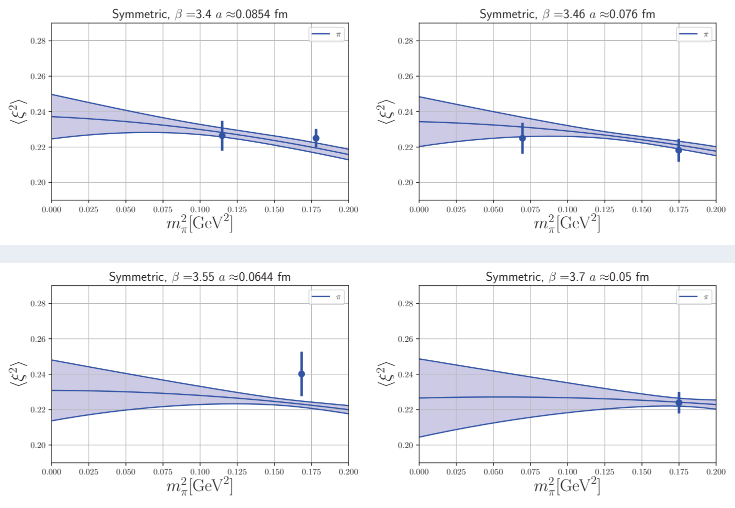
Continuum extrapolation: blue for pion, green for kaon



Continuum extrapolation: blue for pion, green for kaon



Continuum extrapolation: blue for pion, green for kaon



Results

Pion distribution amplitude

Our preliminary result

$$\langle \xi^2 \rangle = 0.236 \pm 0.012$$

is the first ever continuum determination from First Principles. Our previous value at finite lattice spacing and for $N_f = 2$ was 0.236 ± 0.008.

Kaon and eta distribution amplitude

We also measured the kaon first and second moments and can infer from the combined fit the moment of the eta distribution amplitude.

Full x-dependence of the pion DA

In a separate project we are currently estimating non-perturbatively the full x dependence of the pion DA: Braun *et al.* '18

Parton distribution functions from Lattice QCD

- X. Ji (Phys. Rev. Lett. 110, 262002 (2013)) proposed how to recover the light-cone definition of PDF from purely space-like correlations calculable in Lattice QCD,
- In the infinite momentum limit one recovers the light-cone distributions,
- In the framework of Large Momentum Effective Theory one can systematically calculate corrections.

Generic matrix element

Start with a bare matrix element

$$h_\Gamma(P,z) = \langle P|\bar{\psi}(0,z)\Gamma W(z)\psi(0,0)|P\rangle$$

with $|P\rangle$ describing a hadron with momentum P in the direction of the Wilson line $W(z)$, typically

$$P = (P_0, 0, 0, P_3) \qquad z = (0,0,0,z)$$

Hadron structure functions

Quasi-PDF (J.-W. Chen et al, Nucl. Phys. B 2016)

We obtain the Quasi-PDF by an appropriate Fourier transform of the bare matrix element

$$\tilde{q}(x,\Lambda,P) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixPz} h_\Gamma(P,z)$$

The Quasi-PDF has to be matched to QCD

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_3^2}, \frac{M^2}{P_3^2}\right)$$

with $C(\xi, \frac{\mu}{xP_3})$ the matching factor between the LaMET and QCD.

Corrections

- $\frac{\Lambda_{\mathrm{QCD}}}{P_3}$ comes from twist-4 operator, can be parametrized and subtracted
- $\frac{M}{P_3}$ can be estimated to any power and can be taken into account

Nucleon structure functions

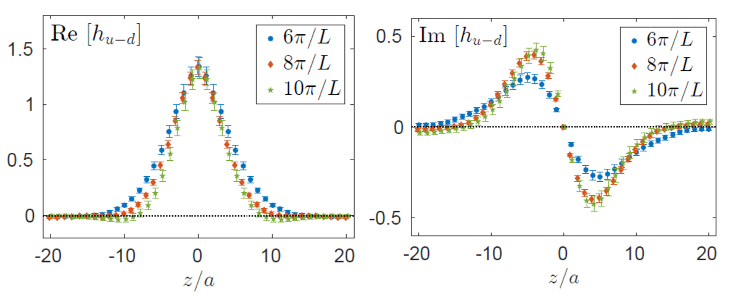
Current status (C. Alexandrou et al., arxiv:1803.02685)

Ensemble:

$$
\begin{aligned}
\beta &= 2.10 \\
c_{\mathrm{SW}} &= 1.57751 \\
a &= 0.0938(3)(2)\ \mathrm{fm} \\
&48^3 \times 96 \\
m_N &= 0.932(4)\ \mathrm{GeV} \\
L &= 4.5\ \mathrm{fm} \\
m_\pi &= 0.1304(4)\ \mathrm{GeV}
\end{aligned}
$$

Nucleon structure functions

Current status (C. Alexandrou et al., arxiv:1803.02685)

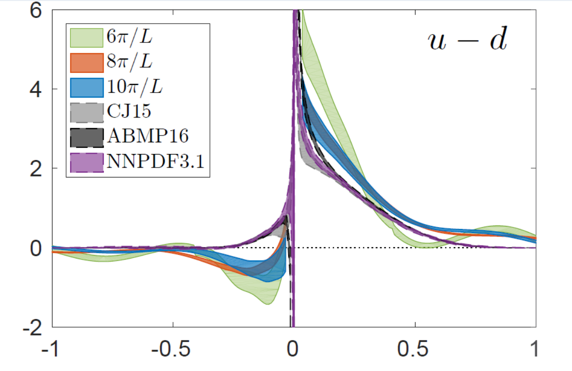


Real and imaginary parts of the bare matrix element for different momenta: $\frac{6\pi}{L} = 0.8$ GeV, $\frac{8\pi}{L} = 1.1$ GeV, $\frac{10\pi}{L} = 1.4$ GeV.

Statistics

$\frac{6\pi}{L}$: 8000 , $\frac{8\pi}{L}$: 40000, $\frac{10\pi}{L}$: 60000.

Current status (C. Alexandrou et al., arxiv:1803.02685)



Unpolarized PDF.

Nucleon structure functions

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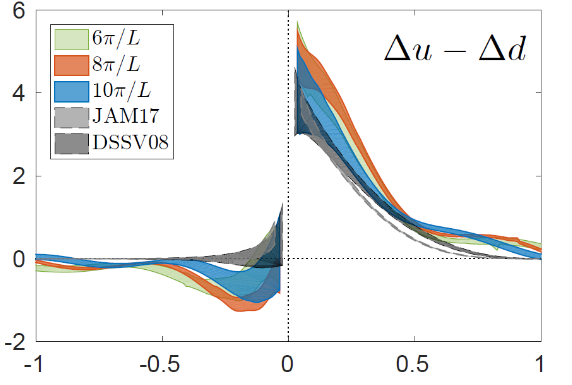
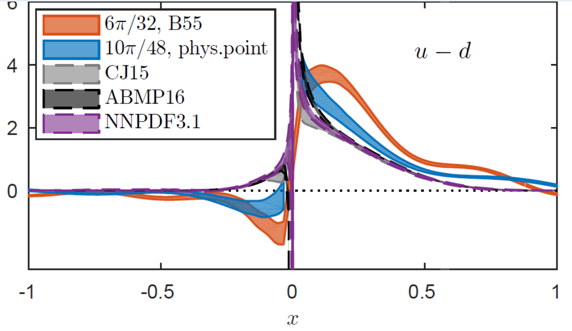


Polarized PDF.

Current status (C. Alexandrou et al., arxiv:1803.02685)



Unpolarized PDF for B55 ensemble: $m_\pi = 375$ MeV and the physical point ensemble.

Pion distribution amplitude: full x-dependence

Problematic Wilson line (Eur.Phys.J. C78 (2018) 217)

Braun advocated replacing the Wilson line by a fermionic line. The approach was tested for the pion DA on a single ensemble.

$$T(p\cdot z, z^2) = \langle 0|[\bar{u}q](z/2)[\bar{q}\gamma_5 u](-z/2)|\pi(p)\rangle$$

where the brackets [] denote operator renormalization in $\overline{\mathrm{MS}}$ scheme and the renormalization scale is fixed to $\mu = 2/\sqrt{-z^2}$.
Using continuum perturbation theory and standard QCD factorization techniques one gets

$$T(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4}\Phi(p\cdot z)$$

and

$$\Phi(p\cdot z) = \int_0^1 du e^{i(u-\frac{1}{2})(p\cdot z)}\phi_\pi(u)$$

Pion distribution amplitude: full x-dependence

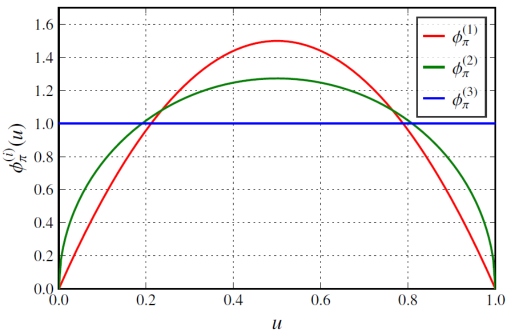
Three models

For illustration we consider three models for the pion DA

$$\phi_\pi^1(u) = 6u(1-u)$$

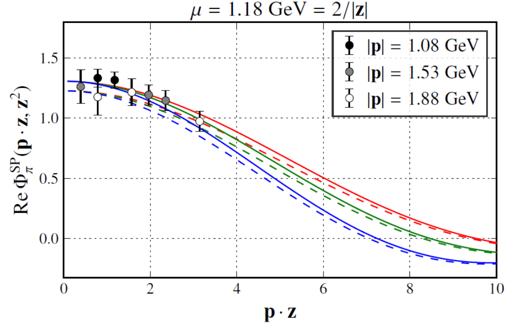
$$\phi_\pi^2(u) = \frac{8}{\pi}\sqrt{u(1-u)}$$

$$\phi_\pi^3(u) = 1$$



Pion distribution amplitude: full x-dependence

Results



Field Programmable Gate Array

- type of processors equipped with ARM cores and programmable logic,
- parts of the program are implemented in hardware
- new kind of parallelizm
- new software developement enviroments allow to write programs directly in C++

First attemps

We (Departement of Discrete Field Theory and Departement of Applied Computer Science, WFAIS, UJ) have implemented the Conjugate Gradient algorithm and use it to invert the Wilson-Dirac operator on a 6^4 lattice. The problem seem to be completely compute bound. We see a speed-up of $\mathcal{O}(10)$ compared to a single core of Intel Xeon Haswell processor. We expect that a speed-up of 50 is achievable.

Conclusions

- Lattice QCD provides non-perturbative, *ab initio* results for hadron structre functions
- systematic effects for the moments of pion distribution amplitude under control
- new method to extract the full x-dependence of structure functions gives good qualitative results
- more computer power needed to push that on a quantitative level

Thank you for your attention!

We acknowledge the Interdisciplinary Centre for Mathematical and Computational Modelling (ICM) of the University of Warsaw for computer time on Okeanos (grant No. GA67-12) and PLGRID and Cyfronet computer center for computer time on the Prometheus supercomputer.