Kaon-kaon interaction in the reactions $e^+ e^- \to K^+K^- \gamma$ and $e^+ e^- \to K^0 \bar{K}^0 \gamma$

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Motivation

1. The S-wave kaon-kaon interaction near the threshold is unknown.
2. Masses of the scalar resonances **f_0(980)** and **a_0(980)** are close to the $K^+ K^-$ threshold.
3. According to the **PDG (2018) estimations**:
   the **f_0(980)** mass = 990 $\pm$ 20 MeV and width= 10 to 100 MeV,
   the **a_0(980)** mass = 980 $\pm$ 20 MeV and width= 50 to 100 MeV.
4. Other parameters of the scalar resonances **f_0(980)** and **a_0(980)** are also imprecise, for example, the branching fractions:
   f_0(980) $\rightarrow \pi\pi$ dominant, **a_0(980)** $\rightarrow \pi\eta$ dominant;
   f_0(980) $\rightarrow K\bar{K}$ seen, **a_0(980)** $\rightarrow K\bar{K}$ seen.
5. The ϕ(1020) meson decays preferentially into the $K^+ K^-$ system in the P-wave. In the radiative decays $\phi \rightarrow M_1 M_2 \gamma$ a pair of pseudo-scalar mesons can be produced in the S-wave.
   The ϕ decay reactions into $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ have been measured, for the ϕ transition into $K^0\bar{K}^0\gamma$ only the upper limit 1.9 10^{-8} is known but there are no data for the $\phi \rightarrow K^+K^-\gamma$.

Motivation to study $e^+ e^- \to K^+ K^- \gamma$

6. There are data for the radiative decay of the $\phi(1020)$ meson into two **scalar resonances**:

$$\Gamma(\phi \to f_0(980)\,\gamma)/\,\Gamma_{total} = (3.22\pm0.19)\,10^{-4},$$
$$\Gamma(\phi \to a_0(980)\,\gamma)/\,\Gamma_{total} = (7.6\pm0.6)\,10^{-5}\,.$$

7. Both scalar resonances decay into the $K^+ K^-$ pairs, so one should observe the reaction $e^+ e^- \to K^+ K^- \gamma$. Here in the final state only the kaon pair interacts strongly.

8. It is not possible to scatter directly kaons on kaons, so the kaon interactions have been studied in the production processes using the pion beams and nucleon targets. Unfortunately the previous studies of the KK interactions in the late seventies and eighties of the XX century gave controversial results.

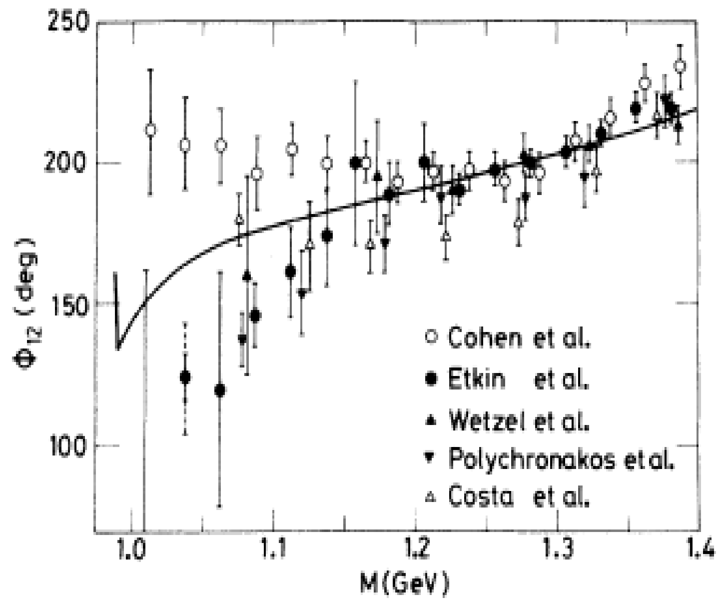
Previous studies of the ππ→KK amplitudes

Reactions: $\pi^- p \to K^+ K^- n$, $\pi^+ n \to K^+ K^- p$, Argonne Lab. 1980 (Cohen et al.)

$\pi^- p \to K^0_S K^0_S n$, Brookhaven Lab. 1982 (Etkin et al.).

Application of the one-pion exchange model to extract the $\pi \pi \to KK$ amplitudes, in particular its S-wave, izospin zero phase Φ_{12}.

Discrepancy between the data for M<1.2 GeV down to the $K \bar{K}$ threshold.



$M = K \bar{K}$ effective mass

Question: is the KK interaction at threshold attractive or repulsive?

Theoretical model of the radiative ϕ decays

1. Our aim: formulation of a general **theoretical model** of the radiative ϕ decays.

2. It should allow for a coupled channel analysis of the amplitudes describing interactions of different meson-meson pairs in the final state.

3. With a help of such a model one can obtain an information on the threshold kaon-kaon S-wave scattering amplitude provided the relevant data are available.

Features of the proposed model

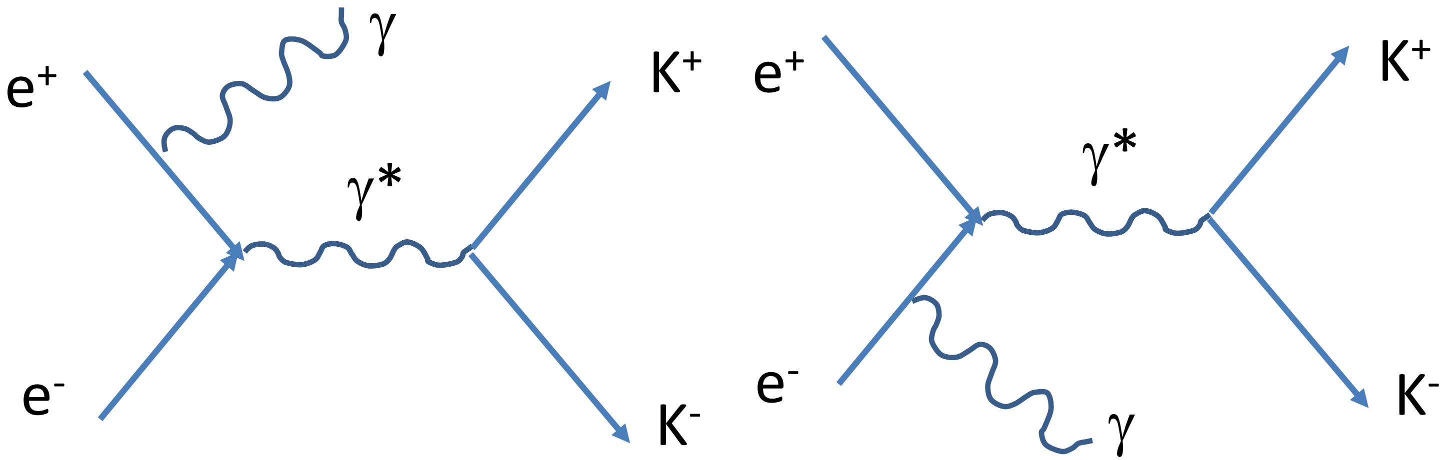
a) **Unitary** description of the coupled channels represented by a set of the transition amplitudes T.

b) **Analiticity** of the amplitudes T in which all the relevant poles corresponding to the scalar resonances are present.

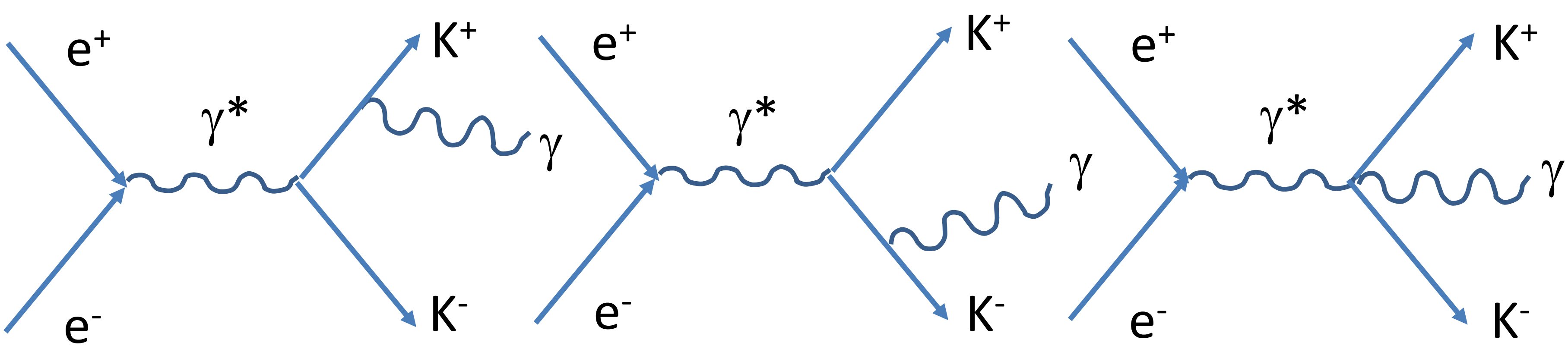
c) Possible application of the model in the combined analyses of many meson-meson final states like:

1. $e^+ e^- \to \pi^+ \pi^- \gamma$,
2. $e^+ e^- \to \pi^0 \pi^0 \gamma$,
3. $e^+ e^- \to \pi^0 \eta \gamma$,
4. $e^+ e^- \to K^0_S K^0_S \gamma$,
5. $e^+ e^- \to K^+ K^- \gamma$.

Reaction mechanisms (1)

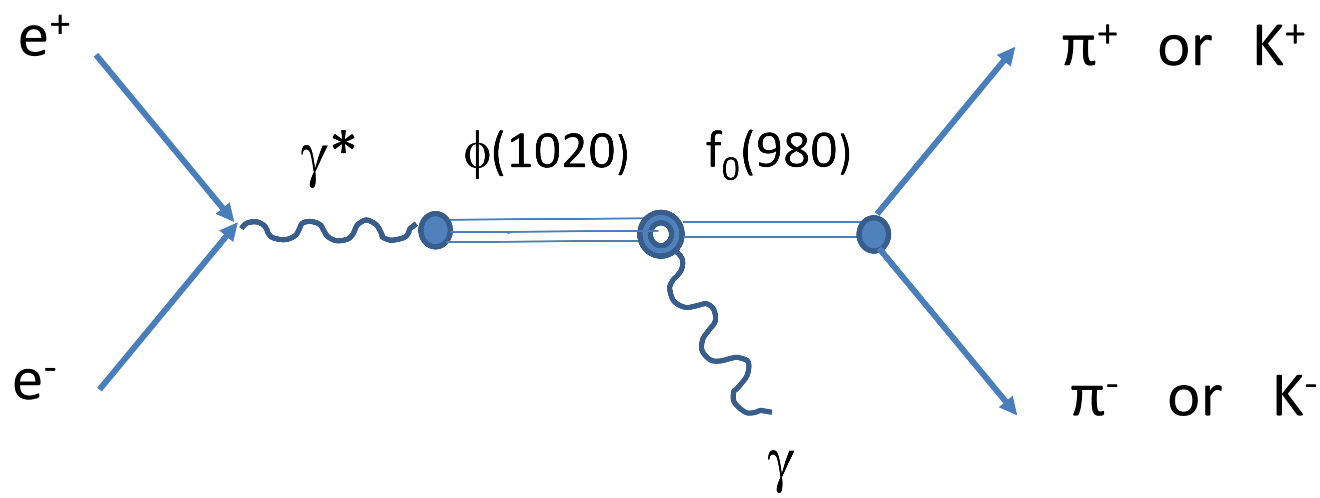


ISR – initial state radiation



FSR – final state radiation

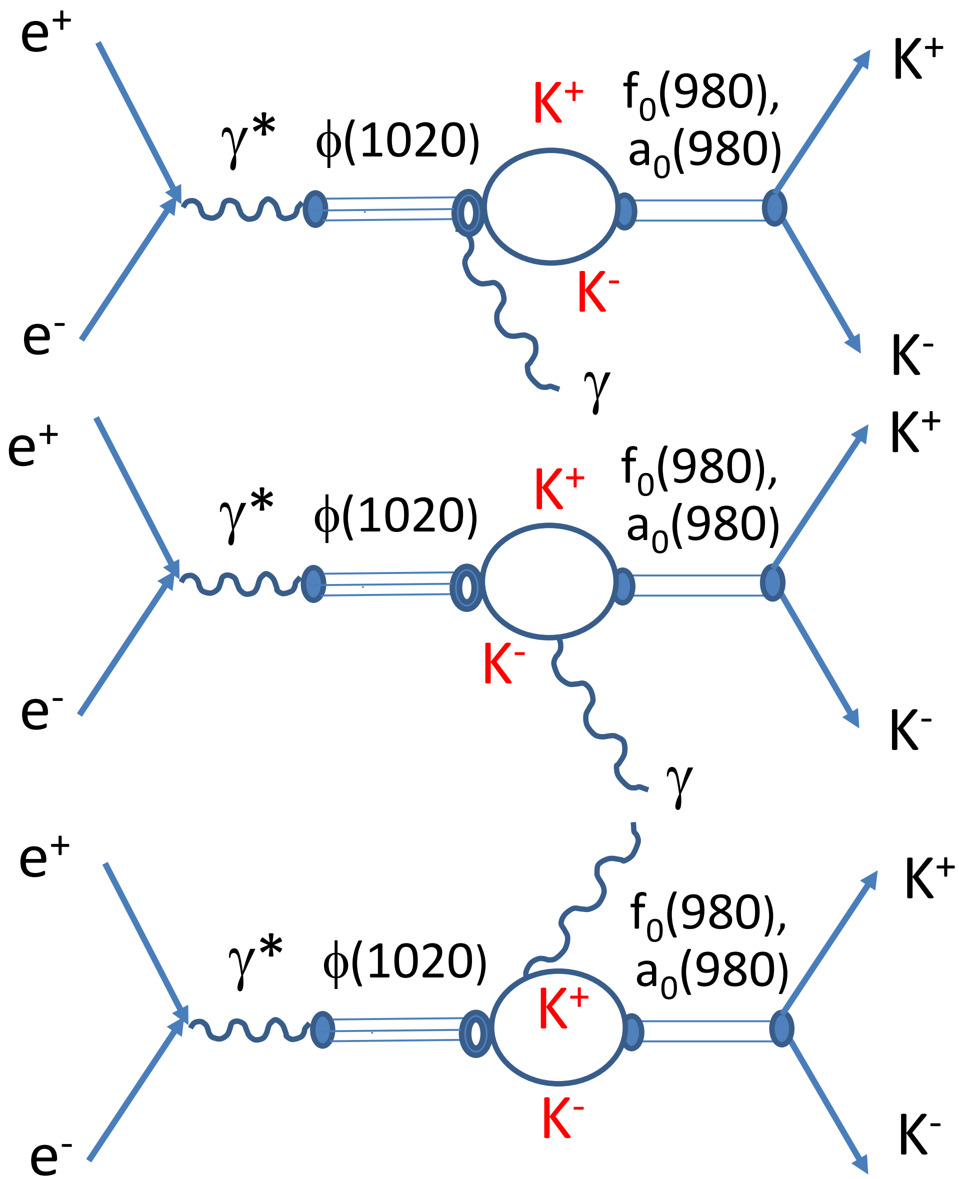
Reaction mechanisms (2)



NS – no-structure model

Ref.: G. Isidori, L. Maiani, M. Nicolaci, S. Pacetti, JHEP 0605 (2006) 049.

Reaction mechanisms (3)



kaon-loop model

Ref.: N. N. Achasov, V. V. Gubin and V. I. Shevchenko, PRD 56(1997)203.

Model dependence

1. Parameters of the scalar resonances found in some experimental analyses are **model dependent**.

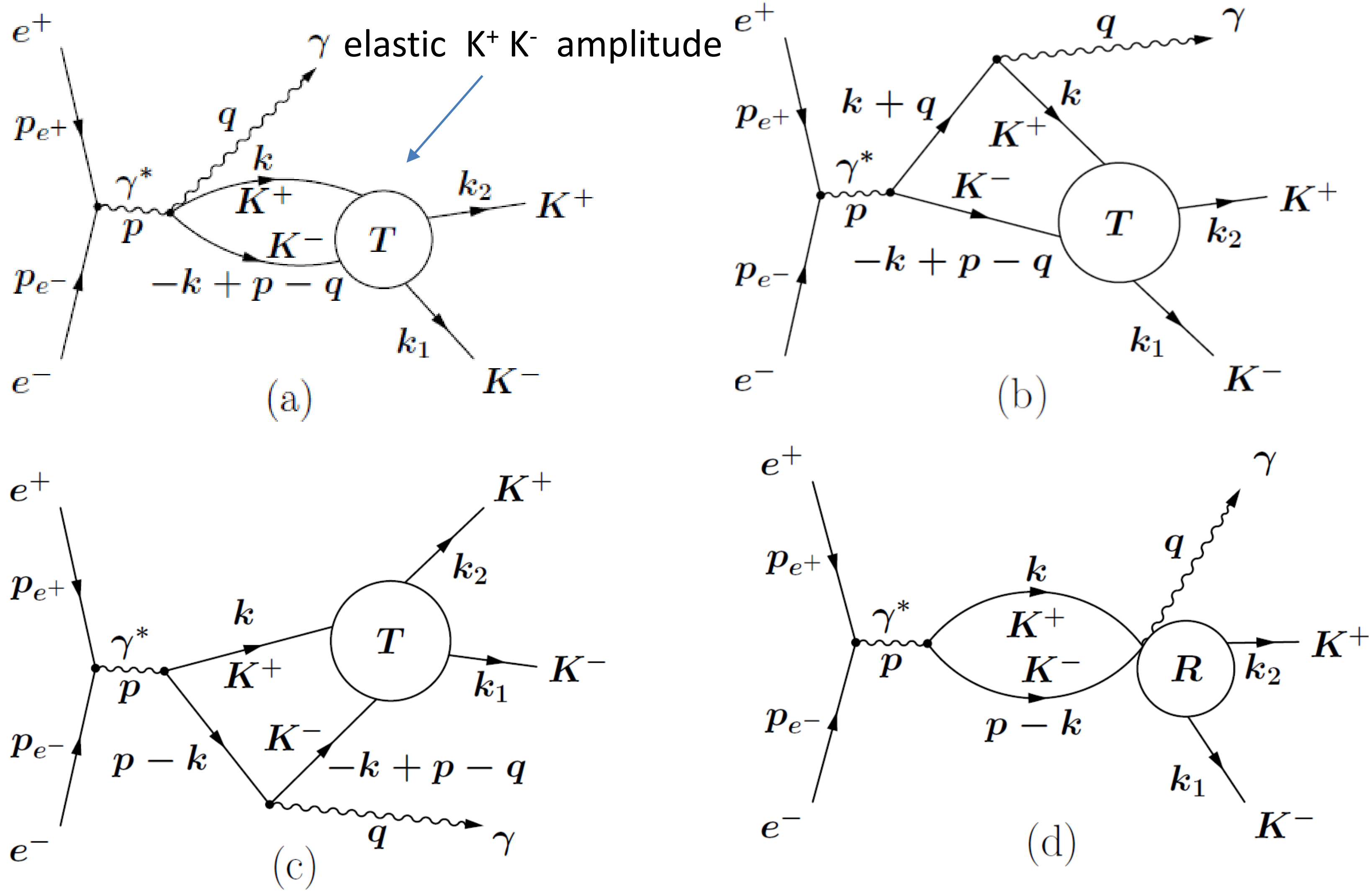
2. Example: reaction $\mathbf{e^+ e^- \rightarrow \pi^+ \pi^- \gamma}$ studied by KLOE in 2006 using two models: kaon-loop (KL) and no-structure (NS).

The results were:

	KL	NS
$f_0(980)$ mass (MeV)	980-987	973-981
$g(f_0 K^+ K^-)$ (GeV)	5.0 – 6.3	1.6 – 2.3
$g(f_0 \pi^+ \pi^-)$ (GeV)	3.0 – 4.2	0.9 – 1.1

g are coupling constants

Diagrams for the reaction $e^+ e^- \to K^+K^-\gamma$



$T(k) = \langle K^-(k_1)\, K^+(k_2) \,|\, \check{T} \,|\, K^-(-k+p-q)\, K^+(k) \rangle$

$R \sim T(k-q) - T(k)$

Amplitudes for the reaction $e^+ e^- \to K^+K^-\gamma$

The total amplitude $A = A_a + A_b + A_c + A_d$ is gauge invariant.

$$A_a = 2i \int \frac{d^4k}{(2\pi)^4} \frac{J_\nu\, \varepsilon^{\nu *}\, T(k)}{D(k)\, D(-k+p-q)}$$

$\varepsilon =$ photon polarization

$$A_b = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu\, \varepsilon^{\nu *} k_\nu (k_\mu + q_\mu)\, T(k)}{D(k+q) D(k)\, D(-k+p-q)}$$

$p = p_{e^+} + p_{e^-}$

$$A_c = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu\, \varepsilon^{\nu *} (k_\nu - p_\nu)\, k_\mu\, T(k)}{D(p-k) D(k)\, D(-k+p-q)}$$

$$A_d = -2i \int \frac{d^4k}{(2\pi)^4} \frac{J \cdot k\, \varepsilon^* \cdot \tilde{k}}{D(k)\, D(p-k)} \frac{[T(k-q) - T(k)]}{q \cdot \tilde{k}};$$

$\tilde{k} = (0, \hat{k});\ \hat{k} = \vec{k} / |\vec{k}|$

$$J_\mu = \frac{e^3}{s} F_K(s) v(p_{e^+}) \gamma_\mu u(p_{e^-})\ ;\ s = (p_{e^+} + p_{e^-})^2$$

$F_K(s)$ – kaon form factor

$$D(k) = k^2 - m_K^2$$

$D(k)$ – inverse of the kaon propagator

Definitions and approximations

$\mathrm{m} = \mathrm{K^+K^-}$ effective mass; $\quad m^2 = (k_1 + k_2)^2$

half – off – shell amplitude: $T(k) = \langle K^-(k_1)\, K^+(k_2) \,|\, \tilde{T}(m) \,|\, K^-(-k+p-q)\, K^+(k) \rangle; \quad k_1^2 = k_2^2 = m_K^2$

on – shell amplitude: $T_{K^+K^-}(m) = \langle K^-(k_1)\, K^+(k_2) \,|\, \tilde{T}(m) \,|\, K^-(p_1)\, K^+(p_2) \rangle, \quad p_1^2 = p_2^2 = m_K^2$

$\mathrm{K^+K^-}$ center-of-mass frame: $\vec{k}_1 + \vec{k}_2 = 0; \quad \vec{p} = \vec{q}; \quad$ photon energy $\omega = |\vec{q}| = \dfrac{s - m^2}{2m}$

relative kaon momentum in the final state: $\vec{k}_{\mathrm{f}} = \dfrac{1}{2}(\vec{k}_2 - \vec{k}_1)$

relative kaon momentum in the initial state: $\vec{k}$

Approximation: $$T(k) \approx g(|\vec{k}|)\, T_{K^+K^-}(m)$$

Condition: $$g(|\vec{k}_f|) = 1$$

$s = m_\phi^2; \quad \omega \le 32\ \mathrm{MeV}$, soft photon

Next approximation: $$\frac{[T(k-q) - T(k)]}{q \cdot \tilde{k}} \approx \frac{g(|\vec{k} - \vec{q}|) - g(|\vec{k}|)}{-\vec{q} \cdot \hat{k}}\, T_{K^+K^-}(m) \approx \frac{dg(|\vec{k}|)}{d\,|\vec{k}|}\, T_{K^+K^-}(m)$$

$\tilde{k} = (0, \hat{k}), \quad \hat{k} =$ unit vector

Approximations, part 2

Dominance of the terms with the **positive** kaon energy $E_k \approx \frac{m}{2}; \quad E_k = \sqrt{|\vec{k}|^2 + m_K^2}$

$$A_a + A_b + A_c \approx \vec{J} \cdot \vec{\varepsilon}^{\,*} T_{K^+K^-}(m)\, I(m)$$

$$I(m) = -2\int \frac{d^3k}{(2\pi)^3} \frac{g(|\vec{k}|)}{2E_k m(m-2E_k)} [1 - 2\frac{|\vec{k}|^2 - (\vec{k}\cdot\hat{q})^2}{2p_0E_k - s + 2\vec{k}\cdot\vec{q}}]; \quad p_0 = m + \omega$$

Limit of vanishing photon energy: $\omega \to 0$, $m \to m_\phi$

$$A_d(0) = -[A_a(0) + A_b(0) + A_c(0)] \qquad \textit{if} \quad |\vec{k}|\, g(|\vec{k}|) \to 0 \ \text{at} \ |\vec{k}| \to \infty$$

$A_d(\omega)$ depends very weakly on ω: $A_d(\omega) \approx A_d(0)$

The full amplitude

$$A(m) \approx \vec{J} \cdot \vec{\varepsilon}^{\,*} T_{K^+K^-}(m)\,[I(m) - I(m_\phi)]$$

Comparison with two other approaches

1. K^+K^- system as a quasi-bound state (kaons as extended objects)

F. Close, N. Isgur, S. Kumano, Nucl. Phys. B 389 (1993) 513

$$g(|\vec{k}|) = \frac{\mu^4}{(\mu^2 + |\vec{k}|^2)^2}; \quad \mu = 141\ \text{MeV}; \qquad \text{note that } g(|\vec{k}| = 0) = 1$$

$$\text{and} \quad g(|\vec{k}| \to 0 \text{ if } |\vec{k}| \to \infty.$$

2. Point-like K^+K^- system

N.N. Achasov, V.V. Gubin, Phys. Rev. D 64 (2001) 094016

$$g(|\vec{k}|) \equiv 1$$

Resonant K^+K^- amplitude:
$R = f_0(980)$ or $a_0(980)$

$$T_{res} = \frac{(c_{RK^+K^-})^2}{D_R(m)}$$

c = coupling constant
D_R = inverse of the propagator R

Remark: **both approaches** can be treated **as special cases** of the present model.

Elastic and transition $K\bar{K}$ amplitudes

Relations to the S-matrix elements:

$$T_{K^+K^-}(m) = \frac{4\pi\, m}{i\, k_f}(S_{K^+K^-} - 1) \qquad T_{K^+K^-\to K^0\bar{K}^0}(m) = \frac{4\pi\, m}{i\, k_f} S_{K^+K^-\to K^0\bar{K}^0}$$

Isospin decomposition: index 0 or 1

$$T_{K^+K^-}(m) = \frac{1}{2}[t_0(m) + t_1(m)] \qquad T_{K^+K^-\to K^0\bar{K}^0}(m) = \frac{1}{2}[t_0(m) - t_1(m)]$$

Example: **separable $K\bar{K}$ interactions in the S-wave**

V=potential
λ=potential strength
β=range parameter
G(k)= Yamaguchi form factor

$$\langle k_f | V | k_i \rangle = \lambda G(k_f) G(k_i)$$

$$G(k) = \sqrt{\frac{4\pi}{m_K}} \frac{1}{k^2 + \beta^2}$$

Separable amplitudes:

off-shell :

$$T_{off} = \langle k_f | T_{sep} | k \rangle = G(k_f)\tau(m)G(k)$$

on-shell :

$$T_{on} = \langle k_f | T_{sep} | k_f \rangle = G(k_f)\tau(m)G(k_f)$$

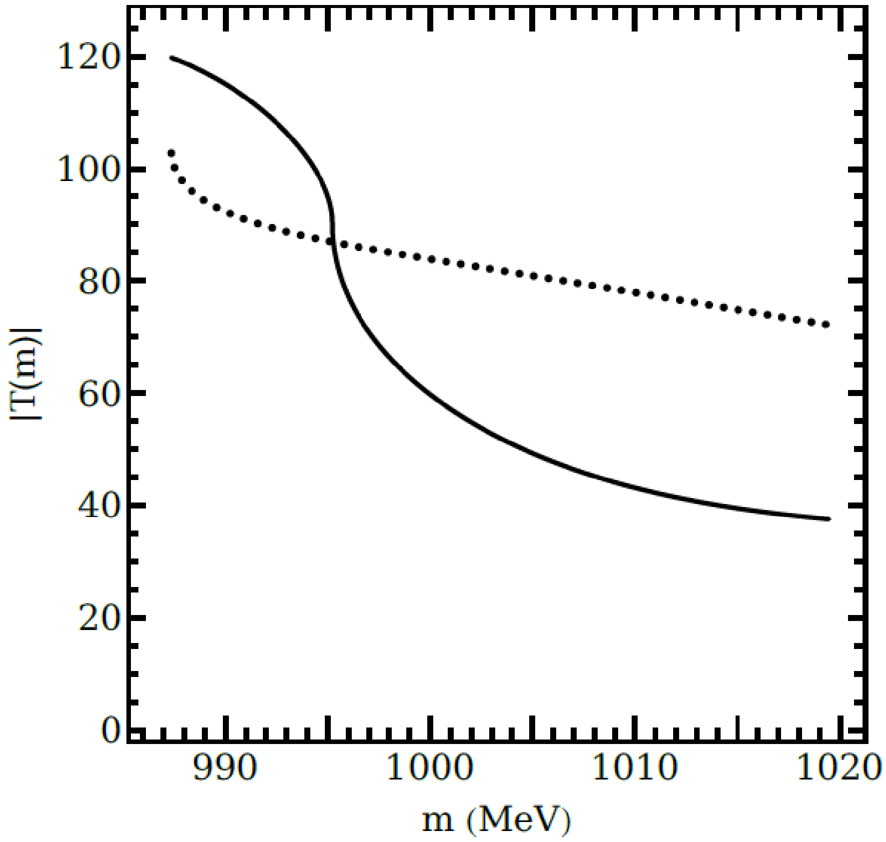
The function g(k)

$$g(k) = \frac{T_{off}}{T_{on}} = \frac{k_f^2 + \beta^2}{k^2 + \beta^2}$$

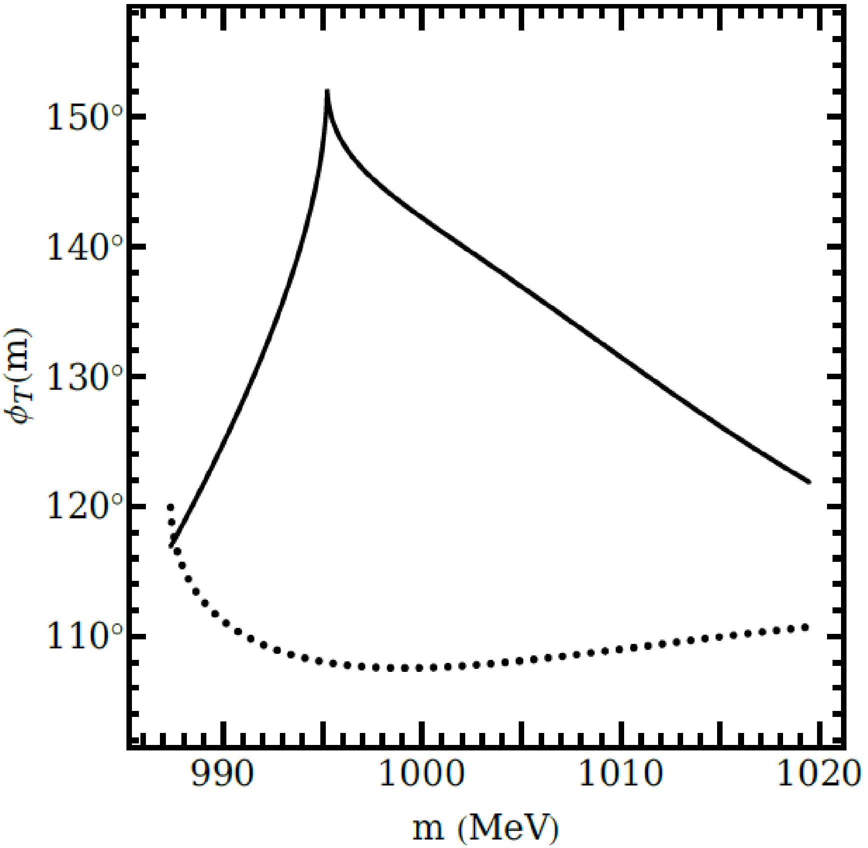
β= 1.5 GeV
for isospin 0

K^+K^- scattering and transition amplitudes

modulus



phase

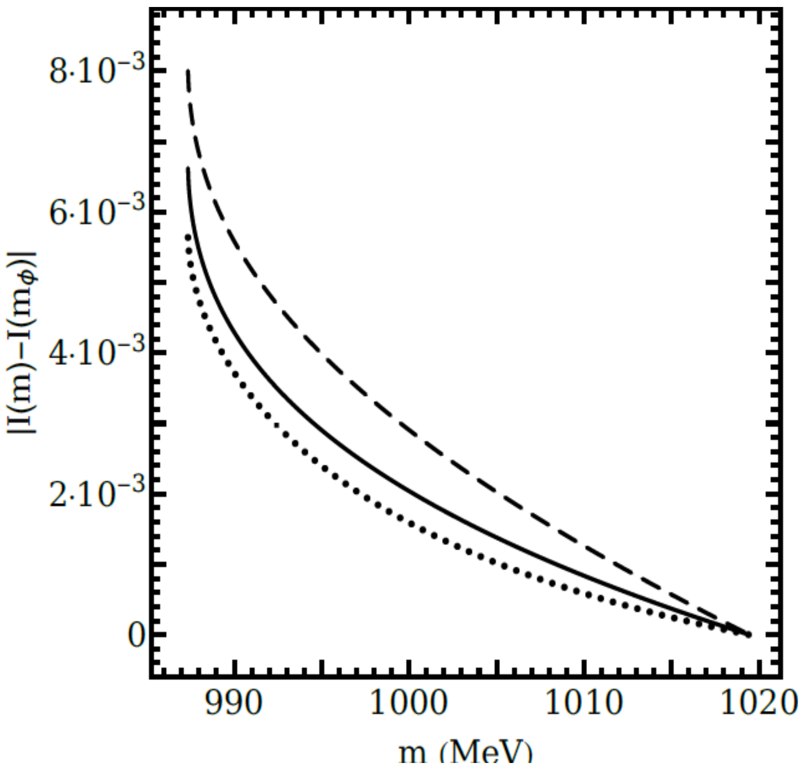


$$\text{dotted line}: K^+K^- \to K^+K^- \text{ elastic amplitude}$$

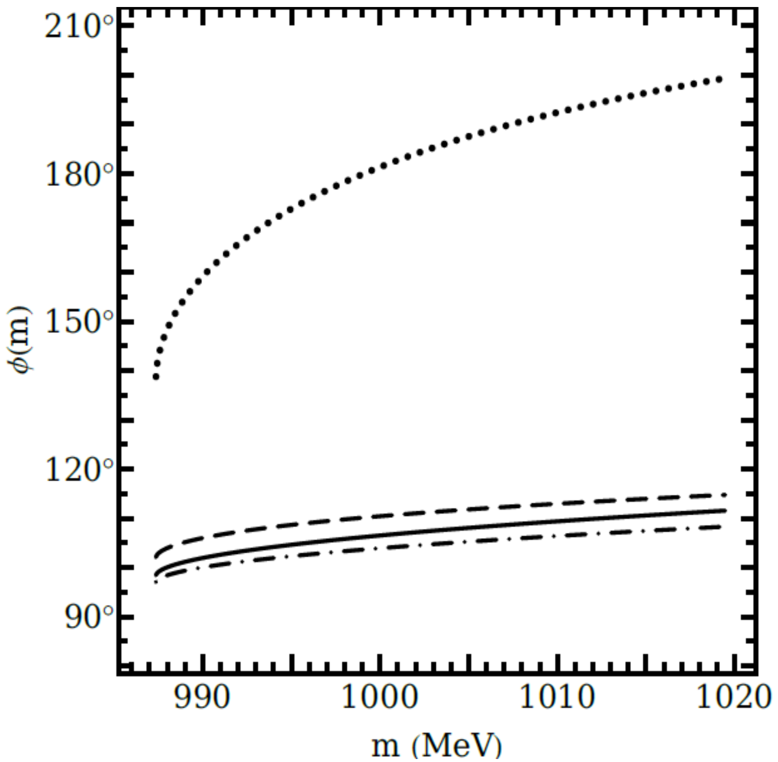
$$\text{solid line}: K^+K^- \to K^0\overline{K}^0 \text{ transition amplitude}$$

The kaon loop function I(m $_\phi$)-I(m)

modulus



phase



solid line: $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and $k_{cut} = 1\,\mathrm{GeV}$

dashed line: $g(k) = \dfrac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$

dotted line: $g(k) = \dfrac{\mu^4}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$

dashed-dotted line: $g(k) \equiv 1$ (point-like case)

Physical observables

Matrix elements squared for the processes $e^+ e^- \to K^+ K^- \gamma$ and $e^+ e^- \to K^0 \bar{K}^0 \gamma$ are proportional to:

$$U(m) = \left(\frac{e^3}{s}\right)^2 |F_K(s)|^2 \, |I(m) - I(m_\phi)|^2$$

Photon angular distribution in the $e^+ e^-$ center-of-mass frame:

$$\frac{d\sigma}{d\cos\theta_\gamma} \propto (1 + \cos^2\theta_\gamma)$$

Effective mass distribution for the process $\mathbf{e^+ e^- \to K^+ K^- \gamma}$

$$\frac{d\sigma}{dm} \propto m^2 k_f U(m) \, |T_{K^+K^-}(m)|^2$$

Effective mass distribution for the process $\mathbf{e^+ e^- \to K^0 \bar{K}^0 \gamma}$

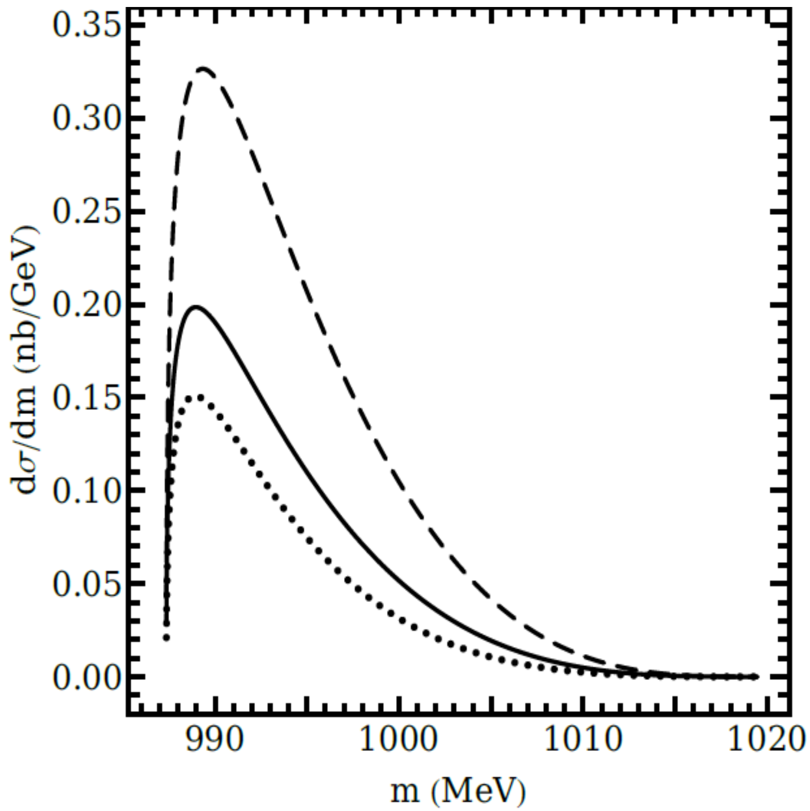
$$\frac{d\sigma}{dm} \propto m^2 k_f U(m) \, |T_{K^+K^- \to K^0 \bar{K}^0}(m)|^2$$

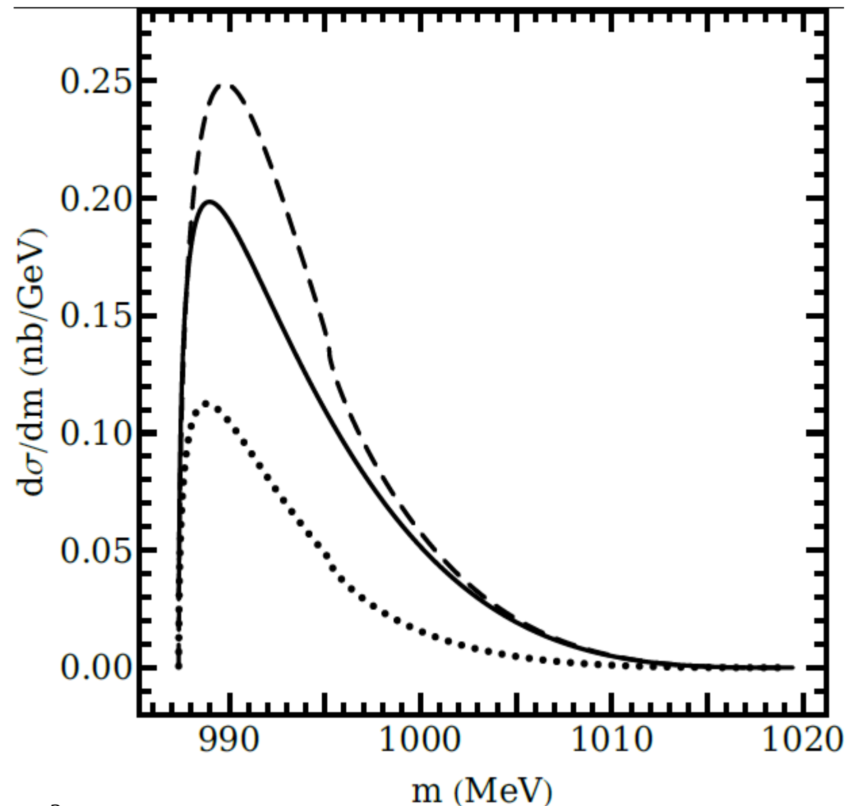
Branching fractions

$$Br(\phi \to K^+ K^- \gamma) = \sigma(e^+ e^- \to K^+ K^- \gamma, \ \mathrm{s} \approx m_\phi^2) / \sigma(e^+ e^- \to \phi)$$

$$Br(\phi \to K^0 \bar{K}^0 \gamma) = \sigma(e^+ e^- \to K^0 \bar{K}^0 \gamma, \ \mathrm{s} \approx m_\phi^2) / \sigma(e^+ e^- \to \phi)$$

Effective mass distributions for $e^+ e^- \to K^+K^-\gamma$





solid line: $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and $k_{cut} = 1\,\text{GeV}$

dashed line: $g(k) = \dfrac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}$, $\mu = 141\,\text{MeV}$

dotted line: $g(k) = \dfrac{\mu^4}{(k^2 + \mu^2)^2}$, $\mu = 141\,\text{MeV}$

dashed line: no-structure model of Isidori et al. (2006)

dotted line: kaon-loop model of Achasov and Gubin (2001)

Transitions into pseudoscalar meson pairs P_1 P_2

Generalization of the model for the process $e^+ e^- \to K^+ K^- \gamma$ to $\mathbf{e^+ e^- \to P_1 P_2 \gamma}$

Inelastic **$K^+ K^- \to P_1 P_2$ amplitude:**

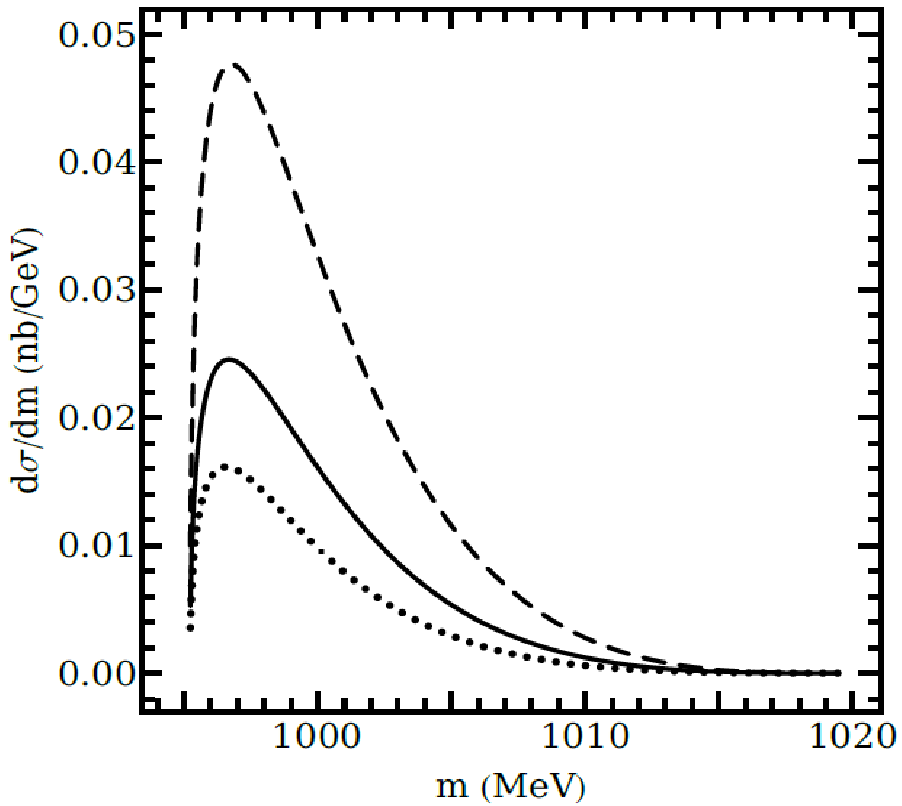
$$T_{K^+K^- \to P_1 P_2} = \frac{4\pi m}{i\sqrt{k_f k_{12}}} \left(S_{K^+K^- \to P_1 P_2} - \delta_{K^+K^-,\, P_1 P_2} \right)$$

k_f = relative $K^+ K^-$ momentum; k_{12} = relative $P_1 P_2$ momentum

Application of the **unitary S- matrix** to several coupled channels:

1. $e^+ e^- \to \pi^+ \pi^- \gamma$
2. $e^+ e^- \to \pi^0 \pi^0 \gamma$
3. $e^+ e^- \to \pi^0 \eta \gamma$
4. $e^+ e^- \to K^0 \overline{K}^0 \gamma$
5. $e^+ e^- \to K^+ K^- \gamma$

Effective mass distributions for $e^+ e^- \to K^0 \bar{K}^0 \gamma$



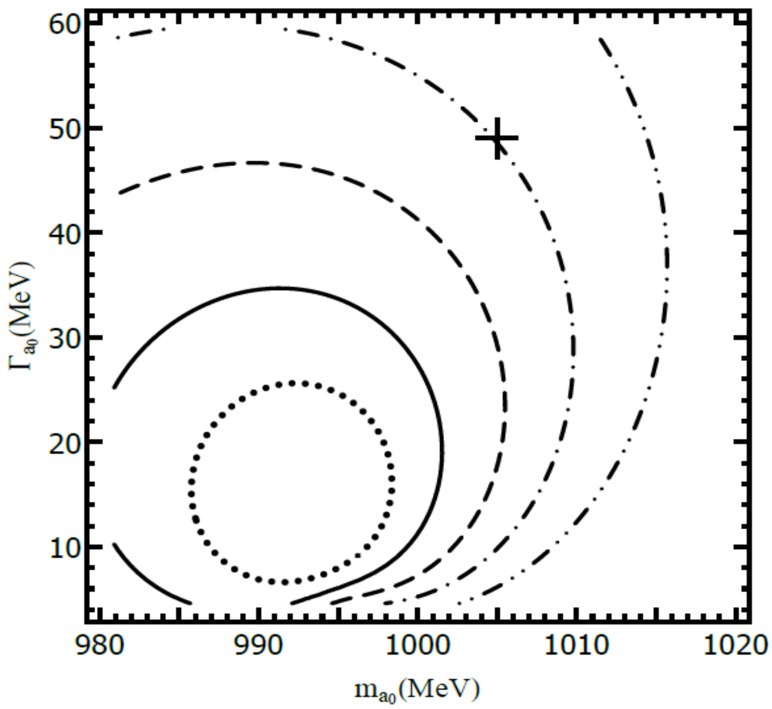
1. solid line: $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$, $\beta = 1.5\,\mathrm{GeV}$ (for I = 0) and $k_{cut} = 1\,\mathrm{GeV}$
2. dotted line: $g(k) = \dfrac{\mu^4}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$, $g(0) = 1$
3. dashed line: $g(k) = \dfrac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$, $g(k_f) = 1$

Total cross sections and branching fractions

	$\sigma_{e^+e^- \to K^+K^-\gamma}(pb)$	$\text{Br}(\phi \to K^+K^-\gamma)$	
1.	1.85	$4.47 \cdot 10^{-7}$	$g(k) = \frac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and k_{cut}= 1 GeV
2.	1.29	$3.10 \cdot 10^{-7}$	$g(k) = \mu^4 / (k^2 + \mu^2)^2$
3.	3.37	$8.13 \cdot 10^{-7}$	$g(k) = (k_f^2 + \mu^2)^2 / (k^2 + \mu^2)^2$
4.	2.29	$5.51 \cdot 10^{-7}$	no-structure model NS
5.	0.85	$2.05 \cdot 10^{-7}$	kaon-loop model KL

	$\sigma_{e^+e^- \to K^0\overline{K}^0\gamma}(pb)$	$\text{Br}(\phi \to K^0\overline{K}^0\gamma)$	
1.	0.167	$4.03 \cdot 10^{-8}$	$g(k) = \frac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and k_{cut}= 1 GeV
2.	0.102	$2.46 \cdot 10^{-8}$	$g(k) = \mu^4 / (k^2 + \mu^2)^2$
3.	0.338	$8.16 \cdot 10^{-8}$	$g(k) = (k_f^2 + \mu^2)^2 / (k^2 + \mu^2)^2$

The $\phi \to K^0 \bar{K}^0 \gamma$ branching fraction and the $a_0(980)$ pole position



Branching fraction of the decay $\phi \to K^0 \bar{K}^0 \gamma$

for $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and $k_{cut} = 1\,\mathrm{GeV}$:

dotted curve	$1 \cdot 10^{-8}$,
solid curve (KLOE limit)	$1.9 \cdot 10^{-8}$,
dashed curve	$3 \cdot 10^{-8}$,
dotted - dashed curve	$4 \cdot 10^{-8}$,
double dotted - dashed curve	$5 \cdot 10^{-8}$

$\Gamma_{a_0(980)}$ = resonance width

$m_{a_0(980)}$ = resonance mass

The values of the branching fraction depend on the **resonance pole position**.

Conclusions

1. A theoretical model of the reactions $e^+ e^- \to K^+ K^- \gamma$ and $e^+ e^- \to K^0 \bar{K}^0 \gamma$ has been formulated.
2. The strong interaction between kaons is taken into account.
3. The elastic $K^+ K^-$ and the transition $K^+ K^- \to K^0 \bar{K}^0$ **amplitudes** in a **general form** can be used.
4. Numerical results for the $K \bar{K}$ effective mass distributions, for the total reaction cross sections and the radiative ϕ decays have been presented.
5. The model can be generalized to treat other coupled channel reactions with **two pseudoscalar mesons** in the final state.
6. Measurements of the $e^+ e^- \to K^+ K^- \gamma$ process can provide a valuable information about the pole positions of the $a_0(980)$ and $f_0(980)$ resonances.