Studies of fluctuations and correlations in high energy heavy ion collisions

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Outline

- ⋆ Introduction
- ⋆ Part 1: Studies of conserved charge fluctuations in Au-Au collisions with CBM
  - CBM experiment
  - Analysis details
  - Results: Net-proton cumulants
- ⋆ Part2: Development of interacting HRG model using s-matrix formalism
- ⋆ Summary

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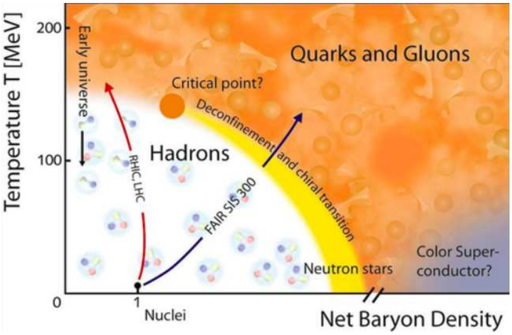
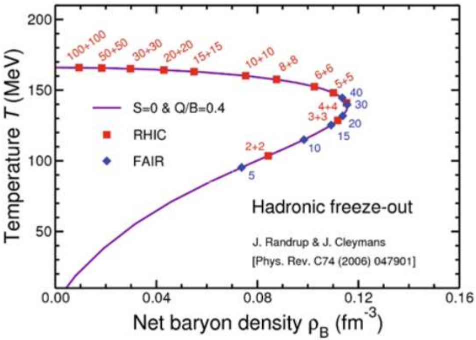
QCD phase diagram





- ⋆ Study QCD phase diagram at high net-baryon density
  - At high net-baryon density and low temperature, first order phase transition is expected which will end at a critical point (CP)
  - CBM program supplements the Beam Energy Scan Program at RHIC, NA61 at SPS, NICA at JINR

Ref: CBM: EPJA 53, 60 (2017); PRC 74, 047901 (2006)

Observables for CP search

Cumulants

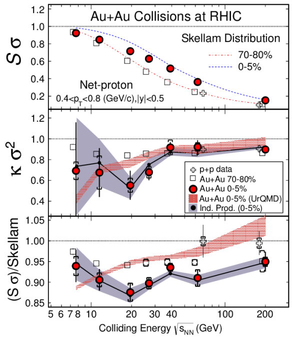
$$C_1 = \langle N_q \rangle, \quad C_2 = \left\langle (\delta N_q)^2 \right\rangle, \quad C_3 = \left\langle (\delta N_q)^3 \right\rangle,$$
$$C_4 = \left\langle (\delta N_q)^4 \right\rangle - 3 \left\langle (\delta N_q)^2 \right\rangle^2$$

$N_q = N_{q^+} - N_{q^-}$ and $\delta N_q = N_q - \langle N_q \rangle$
q can be any conserved quantum number

(net-baryon, net-charge, net-strangeness etc.)

Mean, variance, skewness, kurtosis

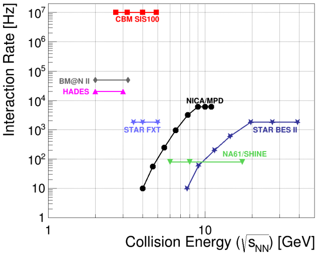
$$M = C_1, \quad \sigma^2 = C_2, \quad S = \frac{C_3}{\sigma^3}, \quad \kappa = \frac{C_4}{\sigma^4}$$



- ⋆ Higher moments of conserved quantities are sensitive to correlation length
  $\left\langle (\delta N_q)^2 \right\rangle \sim \zeta^2 \qquad \left\langle (\delta N_q)^3 \right\rangle \sim \zeta^{4.5} \qquad \left\langle (\delta N_q)^4 \right\rangle \sim \zeta^7$
- ⋆ Non-monotonic variations of $S\sigma = C_3/C_2$, $\kappa\sigma^2 = C_4/C_2$ with beam energy are believed to be good signatures of CP

Ref: STAR: PRL 112, 032302 (2014); PRL 102, 032301 (2009)

CBM experiment



- ⋆ Fixed target experiment
- ⋆ SIS 100: Au + Au collision, $\sqrt{s_{NN}} = 2.7 - 4.9$ GeV
- ⋆ High interaction rate
- ⋆ High statistics data
- ⋆ Density in the center of the fireball expected to exceed few times greater than density of nucleus

Challenges of higher moments measurements at CBM

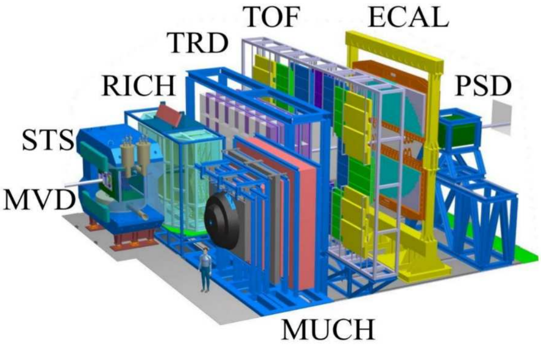
⋆ Particle identification
⋆ Non-trivial variations of efficiency $\times$ acceptance with p_T and rapidity (proper method of corrections needed)
⋆ Proper vertex identification in multiple collisions

Ref: CBM overview talk at QM2019 by Viktor Klochkov; EPJA 53, 60 (2017); The CBM Physics Book, Lect. Notes Phys. 814, Springer 2011; PRC 75 (2007) 034902

Simulation details

- ⋆ Event generators: UrQMD
- ⋆ Collision: Au+Au
- ⋆ Energy: E_{lab} = 10 AGeV ($\sqrt{s_{NN}} = 4.72$ GeV)
- ⋆ Events: 5 M (minimum bias)

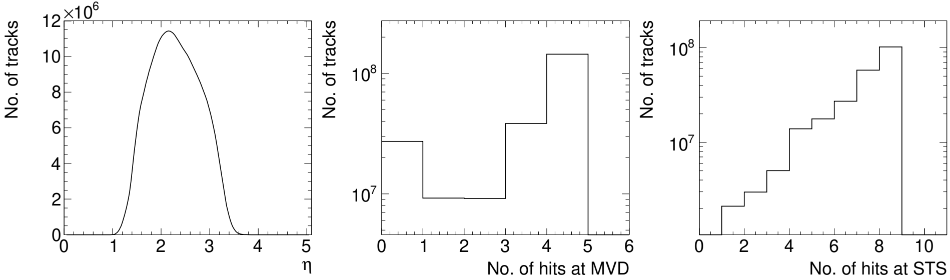
Detectors used:
MVD, STS, RICH, TOF
MVD: Vertex information
STS: Momentum information
RICH: Electron identification
TOF: Hadron identification
Detector acceptance:
$1.5 < \eta < 3.8$ $(25^{\circ} > \theta > 2.5^{\circ})$

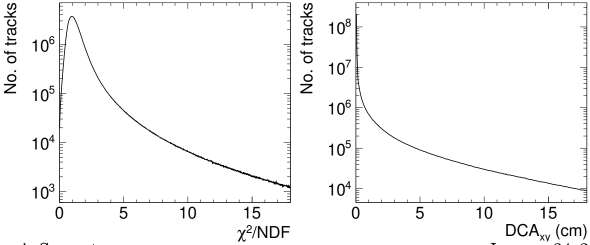


sis100 setup

Track selection

Distributions before applying cuts:





Track cuts

$1.5 < \eta < 3.8$

nHits MVD ≥ 2

nHits STS ≥ 4

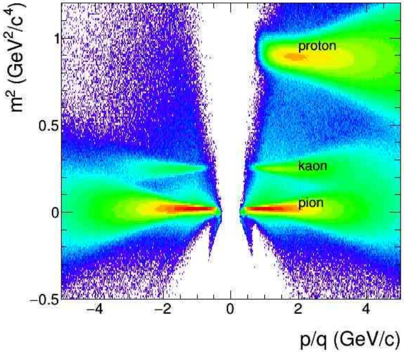
$\chi^2/\mathrm{NDF} < 3$

DCA_xy < 1 cm

Particle identification using TOF

Detector provides time of flight (t)

$$\frac{1}{\beta} = \sqrt{1 + \left(\frac{m}{p}\right)^2} = \frac{tc}{L}, \qquad m^2 = p^2\left(\left(\frac{1}{\beta}\right)^2 - 1\right)$$

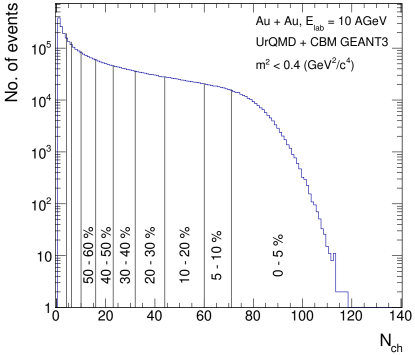


⋆ Clean particle identification for bulk properties studies

- **Centrality detectors:** STS, PSD
  - **Detector used:** STS

To remove auto correlation in net-proton study

- Charge particles selected excluding $p, \bar{p}$
- $m^2 < 0.4$ (GeV^2/c^4)

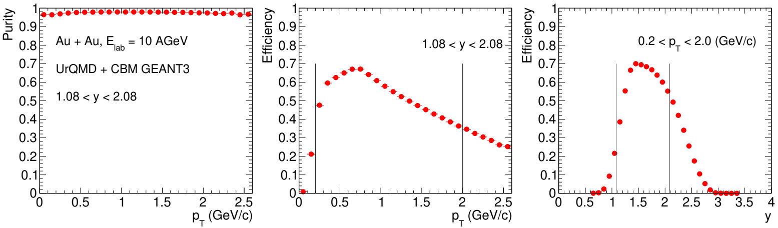


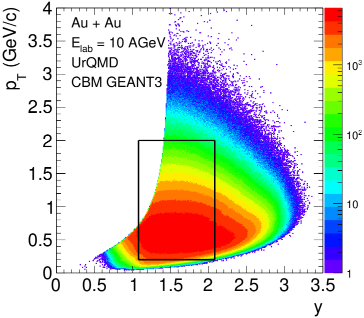
Centrality (%)	N_{ch}
0-5	$N_{ch} \geq 71$
5-10	$60 \leq N_{ch} < 71$
10-20	$44 \leq N_{ch} < 60$
20-30	$32 \leq N_{ch} < 44$
30-40	$23 \leq N_{ch} < 32$
40-50	$16 \leq N_{ch} < 23$
50-60	$10 \leq N_{ch} < 16$
60-70	$6 \leq N_{ch} < 10$
70-80	$4 \leq N_{ch} < 6$

Multiplicities are uncorrected for efficiency and acceptance

Proton (anti-proton) selection

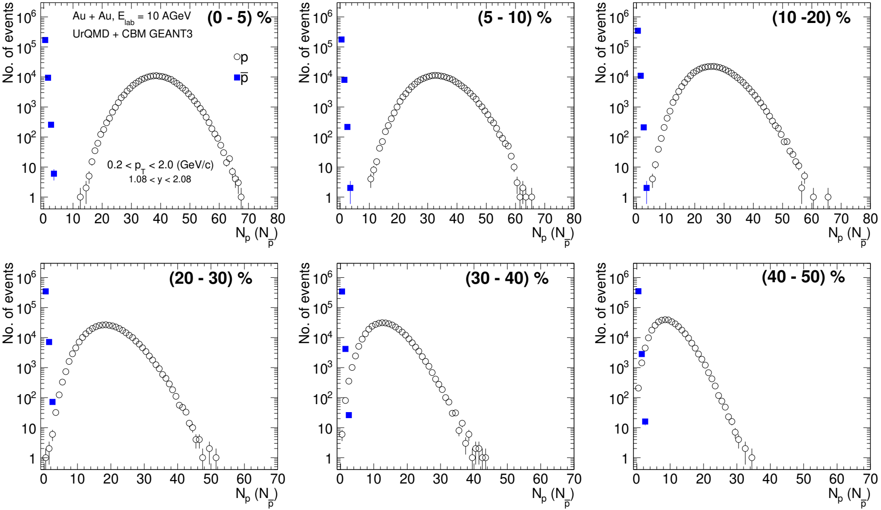
Mass square cut:
$0.6 < m^2 < 1.2\ \mathrm{GeV}^2/\mathrm{c}^4$
Rapidity acceptance:
$\Delta y = 1\ (y_{mid} = 1.58)$
p_T acceptance:
$0.2 < p_T < 2\ \mathrm{GeV/c}$





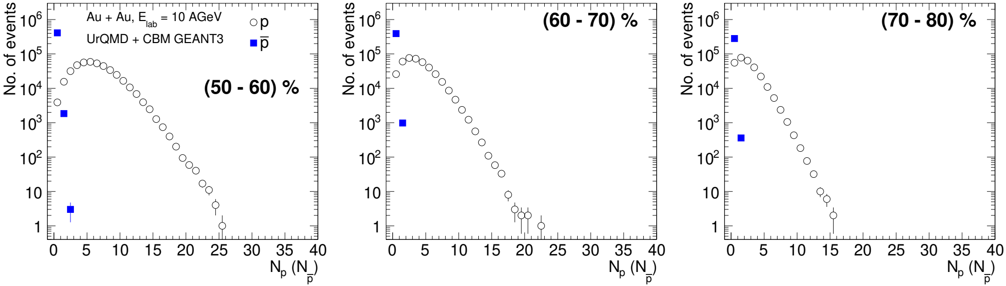
- ⋆ Purity > 96 %
- ⋆ Efficiency decreases at high p_T due to the detector acceptance
- ⋆ Efficiency for 0-5 % and 70 -80 % centralities are $\simeq$ 62 % and $\simeq$ 46 %

Proton (anti-proton) multiplicity distributions



Uncorrected distributions for efficiency and acceptance

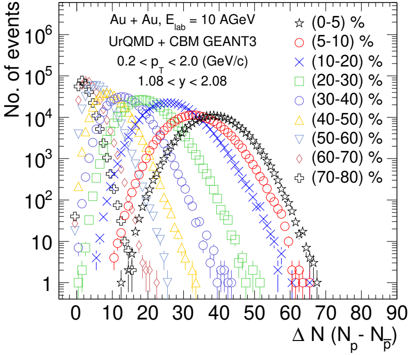
Proton (anti-proton) multiplicity distributions



(Uncorrected distributions for efficiency and acceptance)

- ⋆ Proton multiplicities follow negative binomial distribution
- ⋆ Number of $\bar{p}$ is very less compared to p
  ($\bar{p}/p = 7.8 \times 10^{-5}$ (0 -5 %), $\bar{p}/p = 2.5 \times 10^{-4}$ (70 - 80 %) from UrQMD)
- ⋆ Proton distributions are skewed more to the right side of the mean

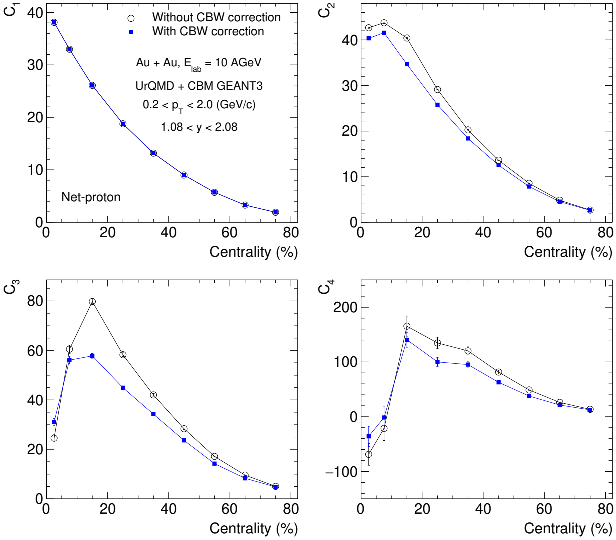
Net-proton multiplicity distributions



Uncorrected for efficiency and acceptance

- ⋆ Mean and variance decreases from central towards the peripheral collisions
- ⋆ Distributions are skewed more to the right side of the mean

C_n of net-proton vs centrality (%)



Centrality bin width correction

$$C_n = \sum_r w_r C_{n,r}$$

$$w_r = \frac{n_r}{\sum_r n_r}$$

$$\sum_r w_r = 1$$

sum is over multiplicity bins

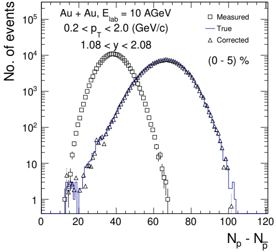
- ⋆ CBWC done to suppress volume fluctuations
- ⋆ Statistical error estimation is done using Delta theorem

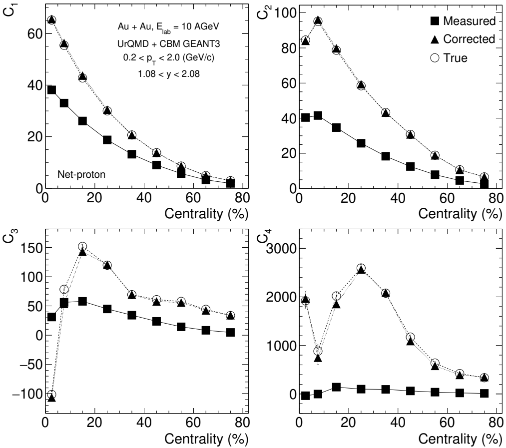
Ref: Advanced Theory of Statistics: Vol.1, London (1945); Asymptotic Theory of Statistics and Probability, Springer (2008); JPG 39, 025008 (2012); JPG 40, 105104 (2013)

Correction of cumulants of net-proton using Unfolding method

Algorithm used: RooUnfoldBayes
Relationship between measured and true distribution: $y = Rx$
y = measured, x = true, R = response matrix

⋆ 50 % events are used to construct R





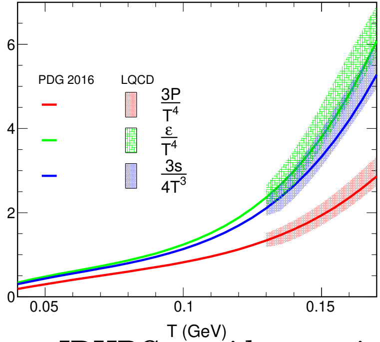
⋆We are able to get back cumulants of 'True', even if the efficiency is non-binomial and has non-trivial dependence on p_T and rapidity

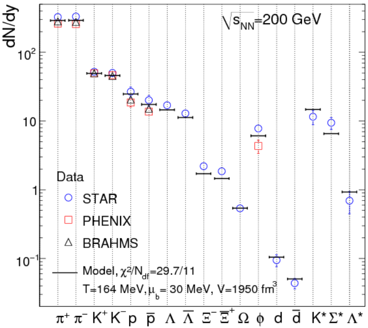
Ref: NIMA 362, 487 (1995)

Ideal Hadron Resonance Gas model

- System consists of all the hadrons including resonances (non-interacting point particles)
- Hadrons are in thermal and chemical equilibrium
- The grand canonical partition function of a hadron resonance gas: $\ln Z = \sum_i \ln Z_i$
- For i th hadron/resonance,
  $$\ln Z_i^{id} = \frac{Vg_i}{2\pi^2} m_i^2 T \sum_{j=1}^{\infty} (\pm 1)^{j-1} (z^j/j^2) K_2(jm_i/T), \quad z = exp(\mu/T),$$
  $$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$
  - The + (-) sign refers to bosons (fermions)
  - The first term ($j = 1$) corresponds to the classical ideal gas
  - Width of the resonances are ignored

Success of IDHRG

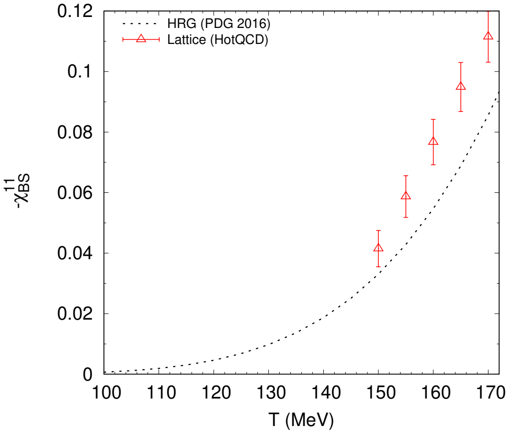


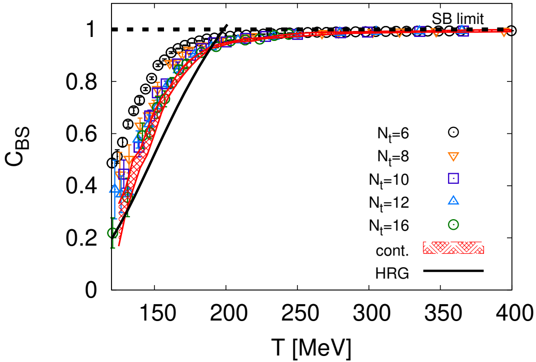


- ⋆ IDHRG provides a satisfactory description in the hadronic phase of continuum LQCD data
- ⋆ Can describe hadronic multiplicities

S. Samanta et al. JPG 46, 065106 (2019); LQCD data: A. Bazavov et al. (HotQCD), PRD 90, 094503 (2014), Andronic, A. et al. PLB673, 142 (2009)

Problem to quantify χ_{BS}, C_{BS}





Ref: A. Borsanyi et al., JHEP01, 138 (2012)

⋆ IDHRG fails to describe χ_{BS}, $C_{BS} = -3\chi^{11}_{BS}/\chi^2_S$

$\Rightarrow$ Interaction is needed

Classical Virial Expansion (Non-relativistic)

$$P = \frac{NT}{V}\left(1 + \left(\frac{N}{V}\right)B(T) + \left(\frac{N}{V}\right)^2 C(T) + ..\right)$$

- The first term in the expansion corresponds to an ideal gas
- The second term is obtained by taking into account the interaction between pairs of particles and subsequent terms involve the interaction between groups of three,four, etc. particles
- $B, C, ...$ are called second, third, etc., virial coefficients

Second virial coefficient

$$B(T) = \frac{1}{2}\int (1 - e^{-U_{12}/T})dV$$

U_{12} is the two body interaction energy

Relativistic Virial Expansion

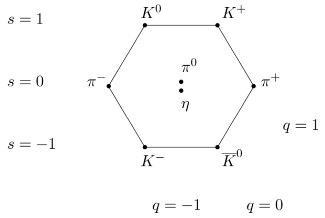
$$\ln Z = \ln Z_0 + \sum_{i_1,i_2} z_1^{i_1} z_2^{i_2} b(i_1,i_2)$$
$$b(i_1,i_2) = \frac{V}{4\pi i}\int \frac{d^3p}{(2\pi)^3}\int d\varepsilon \exp\left(-\beta(p^2+\varepsilon^2)^{1/2}\right)\left[\left\{S^{-1}\frac{\partial S}{\partial\varepsilon} - \frac{\partial S^{-1}}{\partial\varepsilon}S\right\}\right]$$

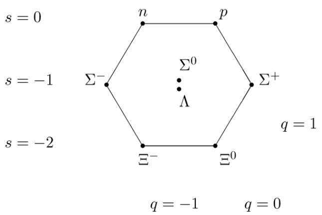
$aa \to R \to aa,\ ab \to R \to ab,\ aab \to R \to aab$ etc.

- ⋆ z_1 and z_2 are fugacities of two species ($z = e^{\beta\mu}$)
- ⋆ The labels i_1 and i_2 refer to a channel of the S-matrix which has an initial state containing $i_1 + i_2$ particles

Second virial coefficient

$b_2 = b(i_1,i_2)/V$ where $i_1 = i_2 = 1$





$\pi\pi \to R \to \pi\pi$
$\pi K \to R \to \pi K$
$KK \to R \to KK$
$\pi N \to R \to \pi N$
etc.

Interacting part of pressure

b_2 in terms of phase shift

$b_2 = \frac{1}{2\pi^3\beta}\int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{l,I}{}' g_{I,l} \frac{\partial \delta_l^I(\varepsilon)}{\partial \varepsilon}$

$$\mathrm{P}_{\mathrm{int}} = \frac{1}{\beta}\frac{\partial \ln Z_{int}}{\partial V} = \frac{1}{\beta} z_1 z_2 b_2$$

$$= \frac{z_1 z_2}{2\pi^3\beta^2}\int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{I,l}{}' g_{I,l} \frac{\partial \delta_l^I(\varepsilon)}{\partial \varepsilon}$$

- ⋆ Interaction is attractive (repulsive) if derivative of the phase shift is positive (negative)

K-matrix formalism (Attractive part of the interaction)

Scattering amplitude: $S_{ab\rightarrow cd} = \langle cd|S|ab\rangle$

Scattering operator (matrix)

$$S = I + 2iT$$

S is unitary

$$SS^{\dagger} = S^{\dagger}S = I$$

$$(T^{-1} + iI)^{\dagger} = T^{-1} + iI$$

$K^{-1} = T^{-1} + iI, \qquad K = K^{\dagger}$ (i.e., K matrix is real and symmetric)

Phase shift in K-matrix formalism

$$\mathrm{Re}\,T = K(I+K^2)^{-1}, \quad \mathrm{Im}\,T = K^2(I+K^2)^{-1} \Rightarrow \mathrm{Im}\,T/\,\mathrm{Re}\,T = K$$

$$K_{ab\to R\to ab} = \sum_R \frac{m_R \Gamma_{R\to ab}(\sqrt{s})}{m_R^2 - s}$$

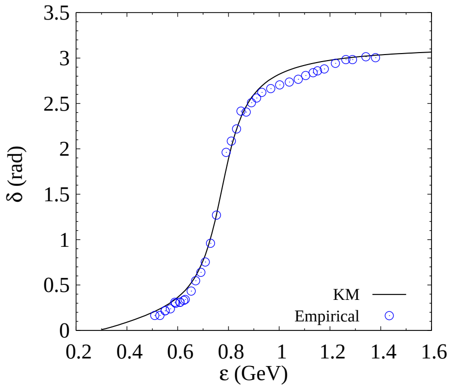
Resonances appear as sum of poles in the K matrix

Partial wave decomposition

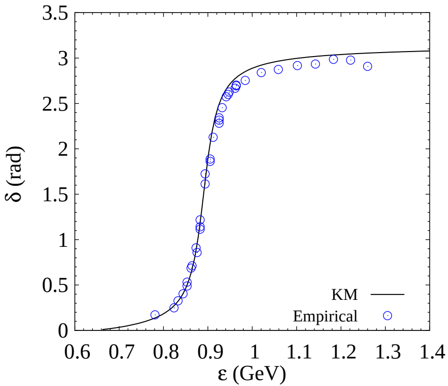
$$S_l = \exp(2i\delta_l) = 1 + 2iT_l$$
$$\Rightarrow T_l = \exp(i\delta)\sin(\delta_l)$$
$$\mathrm{Re}\,T_l = \sin(\delta_l)\cos(\delta_l), \qquad \mathrm{Im}\,T_l = \sin^2(\delta_l)$$

$$K = \tan(\delta_l), \quad \delta_l = \tan^{-1}(K)$$

Phase shift: Empirical vs KM



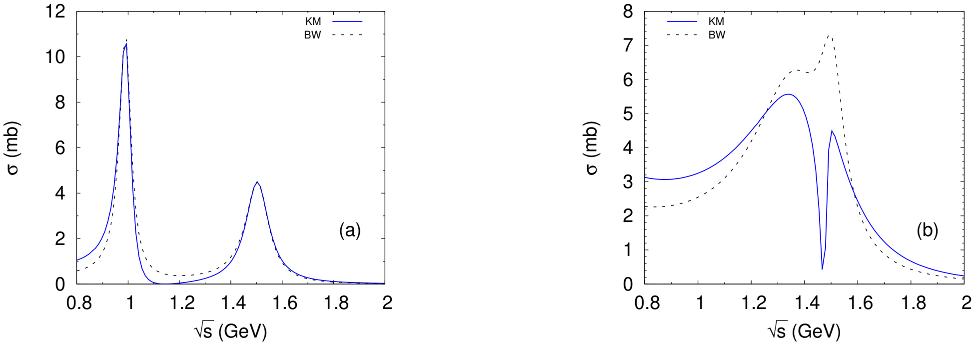
$\pi\pi \to \rho(770) \to \pi\pi$



$\pi K \to K^*(892) \to \pi K$

- ⋆ Good agreement between the empirical phase shifts of resonances and the K-matrix approach

Comparison between K-matrix and Breit-Wigner approach



f_0 (980) (m_1 = 990 MeV, Γ_1 = 55 MeV) and f_0 (1500) (m_2 = 1505 MeV, Γ_2 = 109 MeV)

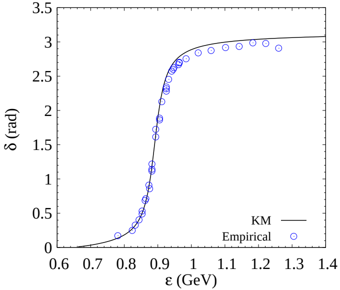
f_0 (1370) (m_1 = 1370 MeV, Γ_1 = 350 MeV) and f_0 (1500) (m_2 = 1505 MeV, Γ_2 = 109 MeV)

- ⋆ KM formalism preserves the unitarity of the S matrix and neatly handles overlapping resonances

S. Samanta et al. PRC 97, 055208 (2018)

Ideal gas limit

- ⋆ For a narrow resonance, δ_l^I changes rapidly through π radian around $\varepsilon = m_R$
- ⋆ δ_l^I can be approximated by a step function: $\delta_l^I \sim \Theta(\varepsilon - m_R)$
- ⋆ $\partial\delta_l^I/\partial\varepsilon \approx \pi\delta(\varepsilon - m_R)$



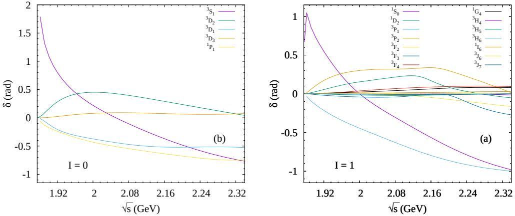
$$b_2 = \frac{1}{2\pi^3\beta}\int_M^\infty d\varepsilon\varepsilon^2 K_2(\beta\varepsilon)\sum_{l,I}{}' g_{I,l}\frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon}$$

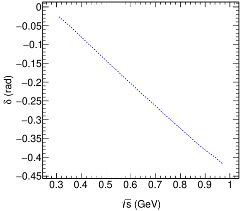
$$= \frac{g_{I,l}}{2\pi^2} m_R^2 T K_2(\beta m_R)$$

$$\mathrm{P_{int}} = T z_1 z_2 b_2 = \mathrm{P_{id}^R}$$

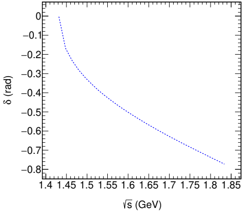
- ⋆ Pressure exerted by an ideal (MB) gas of particles of mass m_R
- ⋆ This establishes the fundamental premise of the IDHRG model

Repulsive interaction from experimental data of phase shift

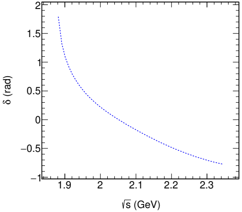




$\pi\pi$: (δ_0^2)



KN : S_{11}



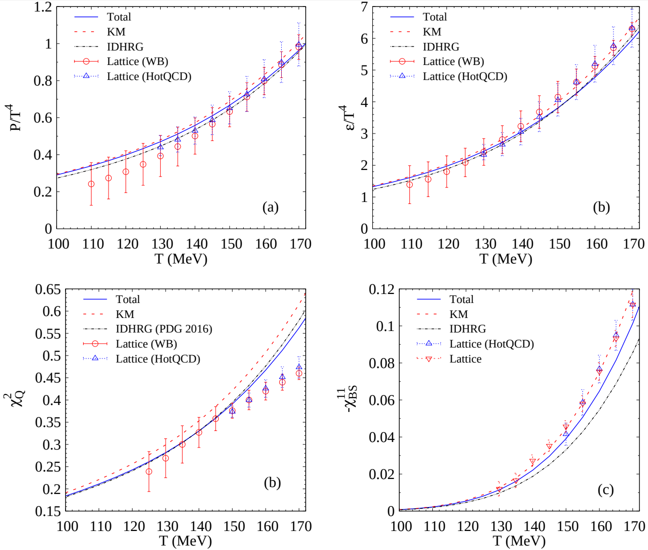
πN: S_{31}

- ⋆ NN interaction: All available data
- ⋆ $\pi\pi$ repulsive interaction: δ_0^2
- ⋆ KN repulsive interaction: $S_{11}(l_{I,2J})$ $(\Sigma(1660))$
- ⋆ πN repulsive interaction: S_{31} $(l_{2I,2J})$ $(\Delta(1620))$, $\Delta(1910)$, $N(1720)$ etc.

- ⋆ $\Sigma(1660)$, $\Sigma(1750)$, $\Sigma(1915)$, $\Delta(1620))$, $\Delta(1910)$, $\Delta(1930)$, $N(1720)$ etc. are included in the repulsive part

Ref: SAID [http://gwdac.phys.gwu.edu]

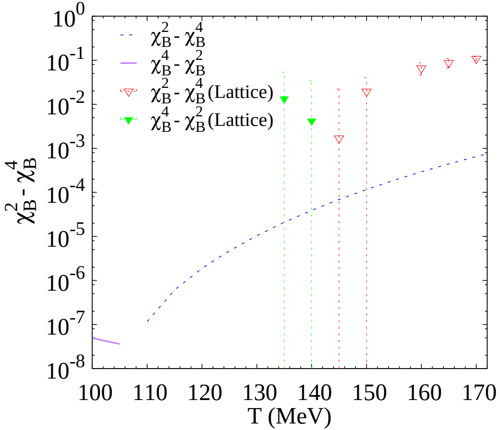
Results

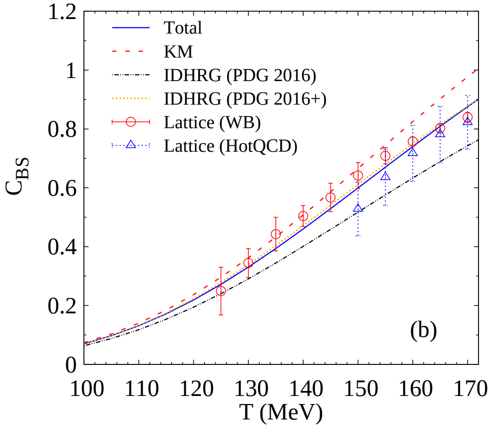


S. Samanta et al. PRC 99, 044919 (2019)

- ⋆ KM: Attractive interaction
- ⋆ Total: Attractive + repulsive
- ⋆ Both KM and Total contain non-interacting part as well
- ⋆ Repulsive interactions suppress the bulk variables

$\chi_B^2 - \chi_B^4$ and C_{BS}

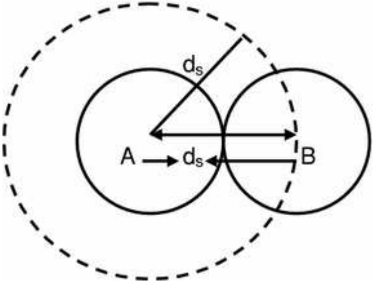




- ⋆ $\chi_B^2 - \chi_B^4$ is non-zero
- ⋆ For C_{BS}: Improvement compared to IDHRG

S. Samanta et al. PRC 99, 044919 (2019)

Excluded volume hadron resonance gas model



- ⋆ Hadrons have finite hard-core radii. ($P(V - Nb) = NT$)
- ⋆ $b = V_{ex} = \frac{16}{3}\pi R^3$ is the volume excluded for the hadron.
- ⋆ Pressure and chemical potential in EVHRG model:

$$P(T, \mu_1, \mu_2, ..) = \sum_i P_i^{id}(T, \hat{\mu}_1, \hat{\mu}_2, ..),$$

$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, ..)$$

van der Waals interaction in HRG model (VDWHRG model)

$$\left(P + \left(\frac{N}{V}\right)^2 a\right)(V - Nb) = NT,$$

$$P(T,n) = \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 \equiv \frac{nT}{1 - bn} - an^2$$

where $n \equiv N/V$ is the number density of particles.

$$P(T,\mu) = P_{id}(T,\mu^*) - an^2, \qquad \mu^* = \mu - bP(T,\mu) - abn^2 + 2an$$

$$n = \frac{n_{id}(T,\mu^*)}{1 + bn_{id}(T,\mu^*)}$$

- ⋆ $a = 0 \quad \Rightarrow$ EVHRG
- ⋆ $a = b = 0 \quad \Rightarrow$ IDHRG

Extraction of parameters a and b

$a = 1250 \pm 150$ MeV fm^3, $r = 0.7 \pm 0.05$ fm

$$\chi^2 = \sum_{i,j} \frac{(R_{i,j}^{LQCD}(T_j) - R_{i,j}^{model}(T_j))^2}{(\Delta_{i,j}^{LQCD}(T_j))^2},$$

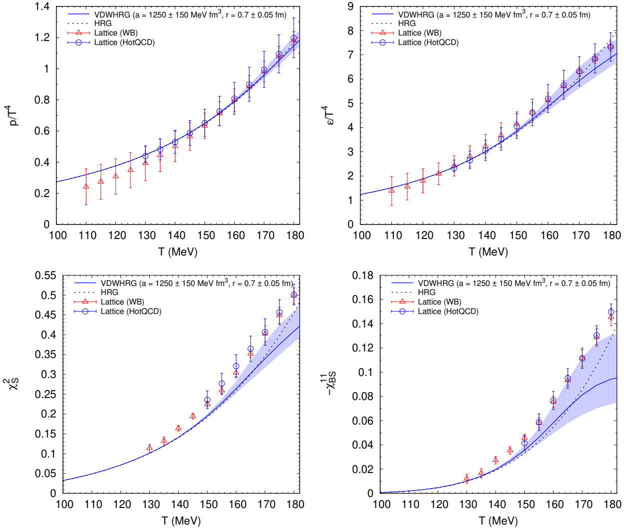
LQCD data of $P/T^4, \varepsilon/T^4, s/T^3, C_V/T^3$ and χ_B^2 at $\mu = 0$ have been used to calculate χ^2

$a = 329$ MeV fm^3, $r = 0.59$ fm

By reproducing the properties of the nuclear matter ($n_0 = 0.16$ fm^{-3}, $E/N = -16$ MeV) at zero temperature

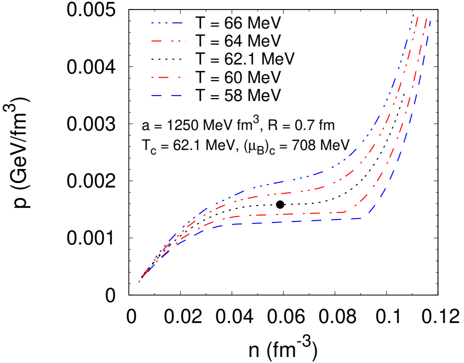
Ref: V. Vovchenko et al., PRC **91**, 064314 (2015)

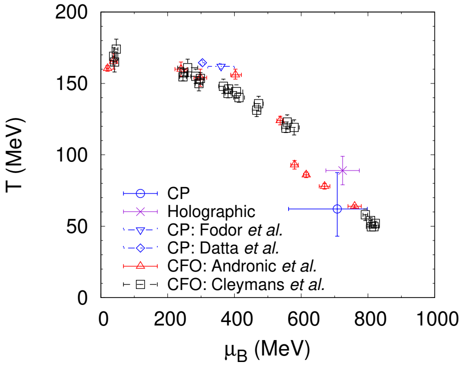
Results- VDWHRG model



S. Samanta et al., PRC **97**, 015201 (2018)

- ⋆ Agreement between LQCD and VDWHRD





- ⋆ Observed first order phase transition
- ⋆ Critical point at $T = 62.1$ MeV, $\mu_B = 708$ MeV
- ⋆ Comparable the CP obtained by using the holographic gauge/gravity correspondence

S. Samanta et al., PRC **97**, 015201 (2018), Holographic: PRD 96, 096026 (2017)

Summary

- ⋆ Studied the feasibility of doing fluctuation analysis with conserved charges in Au+Au collisions at 10 AGeV with CBM detector using simulated events from UrQMD.
- ⋆ Clean proton identification with high purity is possible and hence one can study the net-proton (proxy for net-baryon) higher order moments using CBM detector.
- ⋆ Efficiency and detector effects were corrected for using unfolding techniques and original distributions and cummulants recovered.
- ⋆ An extension of HRG model is constructed to include interactions using relativistic virial expansion of partition function (S-matrix formalism)
- ⋆ Effect of interaction is more visible in $\chi_Q^2, \chi_B^2 - \chi_B^4, C_{BS}$
- ⋆ We find a good agreement for the C_{BS} (without adding extra resonances) and lattice QCD simulations

Thank you