With the help of the table of areas under the normal curve (i.e. - values of the cumulative distribution) solve the following simple problems ($\Phi(x)$ denotes the area under the standard normal curve):

- Given a standard normal distribution, find the area under the curve that lies (a) to the right of $u = 1.84$ (0.0329) , and (b) between $z = -1.97$ and $z = 0.86$. (0.8051 - 0.0244 = 0.7807)
- Given a standard normal distribution, find the value of u such that (a) $P(Z > z) = 0.3015$. (0.52) and (b) $P(k < Z < -0.18) = 0.4197$;

  we have

  $$\Phi(-0.18) - \Phi(k) = 0.4197; \quad \Phi(k) = \Phi(-0.18) - 0.4197 = 0.4286 - 0.4197 = 0.0089$$

  from the table, we have $k = -2.37$
- Given a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

  we standardise:

  $$z_1 = \frac{45 - 50}{10} = -0.5; \quad z_2 = \frac{62 - 50}{10} = 1.2$$

  $P(45 < X < 62) = P(-0.5 < Z < 1.2) = P(Z < 1.2) - P(Z < -0.5) = \Phi(1.2) - \Phi(-0.5) = 0.8849 - 0.3085 = 0.5764$
- Given a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362. we standardise:

  $$z = \frac{362 - 300}{50} = 1.24; \quad P(X > 362) = P(Z > 1.24) = 1 - P(Z < 1.24) = 1 - 0.8925 = 0.1075.$$
- Given a normal distribution with $\mu = 40$ and $\sigma = 6$, find the value of x that has (a) 45% of the area to the left, and (b) 14% of the area to the right.

  From the tables we find $P(Z < -0.13) = 0.45$, so $z = -0.13$. hence $x = (6)(-0.13) + 40 = 39.22$ (we use the standardisation formula ,,backwards"); for (b) – in a similar fashion – $x = 46.48$
- A certain type of storage battery lasts on the average 3.0 years; with a standard deviation of 0.5 year. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

  $$Z = \frac{3 - 2.3}{0.5} = -1.4; \quad P(X < 2.3) = P(Z < -1.4) = 0.808.$$
- An electrical firm manufactures light bulbs that have a length of life that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

  $$z_1 = \frac{778 - 800}{40} = -0.55; \quad z_2 = \frac{834 - 800}{40} = 0.85$$

  $$P(778 < X < 834) = P(-0.55 < Z < 0.85) = P(Z < 0.85) - P(Z < -0.55) = 0.5111$$
- In an industrial process the diameter of a ball bearing must meet the specification: 3.0 ± 0.01 cm. It is known that in the process the diameter of the ball bearing has a normal distribution with mean 3.0 and standard deviation $\sigma = 0.005$. On the average; how many ball bearings will be scrapped?

  Answer: Convert 2.99 and 3.01 into z_1 and z_2; on the average, 4.56% of manufactured ball bearings.
- On an examination the average grade was 74 and the standard deviation was 7. If 12% of the class are given A's; and the grades follow a normal distribution what is the lowest possible A and the highest possible B?

  Answer: we require a z-value that leaves 0.12 of the area to the right and hence an area of 0.88 to the left. From the tables $z = 1.175$ So $x = (7)(1.175) + 74 = 82.225$. Therefore the lowest A is 83 and the highest B is 82.

- For the problem above find the sixth *decile*, i.e. the $q_{0.6}$ quantile. (This is the x-value that leaves 60% of the area under the normal curve to the left.)

  from the tables $z_{0.6} = 0.25$ so $x = (7)(0.25) + 74 = 75.75$. It means that 60% of the grades are 75 or less.

Exponential probability density function

- Suppose the length of time an electric bulb lasts, X, is a random variable with cumulative function

$$F(x) = \mathcal{P}(X \leq x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-x/500} & x \geq 0. \end{cases}$$

  Find the probability that the bulb lasts: (a) between 100 and 200 hours$\mathcal{P}(100 \leq X \leq 200) = F(200) - F(100) = 0.1484$

  (b) beyond 300 hours. $\mathcal{P}(X \geq 300) = 1 - F(300) = 0.5488$

  (c)Find the expected value of the bulb life-time – 500 hours

- Let X be the random variable representing the length of a telephone conversation. Let $f(x) = \lambda e^{-\lambda x}, \quad 0 \leq x < \infty$. Find the c.d.f $F(x)$ and find $\mathcal{P}(5 < X < 10)$.

$$F(X) = \int_0^x f(t)\,dt = 1 - e^{-\lambda x}; \qquad \mathcal{P} = e^{-5\lambda} - e^{-10\lambda}$$